CRITICAL STATE SOIL MECHANICS

- Prof. E.O.E. Pereira Commemorative Inaugural Lecture -

by

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1.0 Introduction

Soil is not only a foundation material that supports structures but also a construction material used for the construction of earth dams, highway and railroad embankments etc. It can be considered as the oldest and the most complex of the construction materials used by engineers.

The term "Soil Mechanics" designates the discipline of engineering science which deals with the properties, behaviour and performance of soil as a structural material. Soil mechanics is one of the youngest disciplines in civil engineering. It found its birth as a scientific discipline with the publishing of the book "Erdbaumechanik" in German by Karl Terzaghi in 1925. Karl Terzaghi is considered as the father of soil mechanics by the civil engineers.

During an address to the Building Research Congress in United Kingdom in 1951, Terzaghi had this to say: "On account of the fact that there is no glory attached to the foundations, and that the sources of success or failure are hidden deep into the ground, building foundations have always been treated as step children; and their acts of revenge for the lack of attention can be very embarrasing".

Civil engineers put great effort into the planning, design and construction of portions of the structures lying above the ground, as these are the parts of the structures that are seen and appreciated by the people. The foundations lie hidden in the ground and therefore tend to get ignored. Hence we see a number of structures around the world which stand as embarrassment to civil engineers due to unsatisfactory foundation design. Two such cases which have become tourist attractions are given below.

The Leaning Tower of Piza in Italy is a bell tower about 179 ft. high, built during the 12th century. The tower was supported by shallow foundations about 6 ft. deep. The sub-strata below the tower foundations is sand but pockets of clayey material are found within the sand layers. This has produced settlement and tilt in the tower, and at present, the top of the tower has moved about 18 ft. from the centre line. Last year, 800,000 visitors climbed up the 294 spiral steps to the top of the tower. Measurements made in June, 1988 shows that the tower has moved further by 1.29 mm (0.05 inch) during the previous 12 months. At the present rate of movement, it is estimated that the tower will topple over in about 100 years.

The Palace of Fine Arts in Mexico city was built in 1904. The sub-strata in this site consists of very deep layers of alluvial deposits of clay, silt and sand. The building was supported on a raft foundation. At present, the building as a whole has settled by about 9 ft, while the differential settlement between parts of the structure is only about 6 inches. The building is in service now with the original ground floor functioning as the basement.

Engineering journals often report on allegedly lawless behaviour of soils that has caused immense damage to life and property. Many more accidents remain hidden in the memories of engineers who have produced unsuccessful designs. These and many other unknown facts concerning the soil induced progressive engineers all over the world to carry out research to understand the behaviour of soils.

In 1775, Coulomb suggested the following relationship for shear strength along a plane of failure in the soil mass:

$$s = C + \sigma \tan \phi \quad \ldots\ldots\ldots (1)$$

where s is the shear strength, c is the cohesion, ϕ is the angle of internal friction and σ is the normal stress.

Terzaghi, in 1925, introduced the concept of effective stress which states that

$$\sigma' = \sigma - u \quad \ldots\ldots\ldots (2)$$

where σ' is the effective normal stress, σ is the total normal stress and u is the pore water pressure. Using this concept, Coulomb's original equation was modified to

$$s = c + \sigma' \tan \phi \quad \ldots\ldots (3)$$

At present, solutions to many soil engineering problems are obtained using the modified Coulomb's equation.

In 1886, Reynold proposed that all soils and other granular media were dilatant and changed in volume when subjected to shear distortion. The significance of this property in understanding the shear behaviour of soils was not appreciated for many years. Hvorslev and Rendulic, around 1936, were the first people to correlate the shear behaviour of soils with dilatation. Hvorslev showed that the shear strength of a soil was a function of the effective normal stress across the plane of failure and the voids ratio in the plane of failure, at the moment of failure. Rendulic showed that a unique relationship exists between effective stresses and voids ratio for a saturated remoulded clay.

The significance of Hvorslev's and Rendulic's work was not appreciated until about the 1950s, when carefully planned research was started in University of Cambridge and Imperial College of Science & Technology. During the last forty years, major contributions have been made in many parts of the world towards the understanding of the stress deformation behaviour of soils.

The main soil engineering problems encountered in practice are generally grouped into two classes; one class known as <u>stability problems</u> and another class known as <u>settlement problems</u>. The class of stability problems includes three main categories, viz. earth pressure problems, slope stability problems, and bearing capacity problems. Stability problems are solved at present by making the assumption that the soil is a rigid plastic material. The soil is assumed to remain rigid up to a certain level of loading, and then to yield suddenly with very large deformation which does not merit calculation. Usually a factor of safety is allowed on this load in order to determine the safe design load.

The deformation of the soil under applied loads is generally calculated only in the case of settlement problems. Stresses due to applied loads are evaluated making the assumption that the soil is a linear elastic material, and settlements are estimated using these stresses.

A practising engineer would ideally like to have deformations that take place in soil mass under existing loads and prior to attaining the working loads. This will only be possible if satisfactory constitutive relationships can be established to describe the shear behaviour of soils. Further, recent developments made in the field of finite element analysis cannot be used advantageously for solving soil engineering problems unless suitable constitutive relationships are available. This approach will provide the deformation occurring in the soil mass at working loads and at failure with the same analysis.

During the last three decades, research workers in soil mechanics have attempted to establish constitutive relationships for soils. Simplifying assumptions have been made in order to derive these relationships. Two basically different approaches have been developed. In the approach developed by Roscoe and his co-workers at Cambridge, the soil is assumed to be an elasto-plastic continuous medium while the other approach developed by Rowe and his co-workers at Manchester is based on particulate mechanics. Extensive research into shear behaviour of soils has been carried out in both Cambridge and Manchester, and constitutive relationships have been developed.

The critical state soil mechanics described herein is based on a soil model called Cam clay model developed by the Cambridge soil mechanics group in the early sixties, and the modifications made to the model thereafter. In this model the soil structure is assumed to behave mechanically as an elasto-plastic isotropic continuous medium. Further, the section of the yield surface for soils is assumed to be of the extended von Mises type. Drucker, Gibson and Henkel (1957) suggested that the yield surfaces are a family of hemispheres expanding to give greater diameters for higher consolidation pressures. In the Cam clay model, it is suggested that the yield surfaces are closer to being the family of bullet shapes. The shape of bullet corresponds to the virgin consolidation. The model has been developed using carefully established experimental data available from standard triaxial tests and then extended to the general stress system.

2.0 Triaxial Test

The standard triaxial test in which two principal stresses remain equal has become the most widely used test to study the shear behaviour of soils both in commercial and in research laboratories. In this test, the deviator stress in the sample is estimated by making the assumption that the sample remains a right circular cylinder during the shearing. The change in voids ratio in the portion of a saturated sample that deforms is estimated by measuring the volume of water that enters or leaves the boundary of the sample and assuming that the deformation in the sample is uniform throughout its length. Roscoe, Schofield and Thurairajah (1963) present data in which axial and radial strains have been measured throughout the length of the sample during standard triaxial compression and extension drained tests on sand. At large axial strains, the behaviour of the triaxial sample during the shearing process is found not to be uniform as assumed. Deformations during triaxial compression tests are more uniform than during triaxial extension tests. Roscoe, Schofield and Wroth (1959) have argued that the deformation during a triaxial compression test is more uniform in a sample that "work-hardens" during the shearing process than in the one that "work-softens". In a sample that "work-hardens", the portion of the sample

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(1964) have shown the existence of a unique surface for all virgin consolidated clay to which all loading paths irrespective of the drainage conditions followed. ABCD in Fig. 2 represents the section of the state boundary surface by an e = constant plane. The curved line AD is the loading path followed by an undrained test on a sample of virgin consolidated clay. When the loading path reaches the point D, shear distortion takes place without change in stress or pore water pressure, and the state of the sample continues to remain at D. The portion of the surface XXMM is a ruled surface. CD is a straight line and the intercept BC on the p = 0 plane increases with decrease in e. The loading path for a heavily over-consolidated sample of clay rises from the q = 0 plane, lying initially inside the state boundary surface, and reaches the ruled part of the surface. The path thereafter moves on this surface and ends on the line XX at the critical state. Roscoe et al. (1958) have shown the existence of the state boundary surface and critical state line for cohesionless material.

6.0 Energy Equation

Consider a triaxial sample of soil which is in equilibrium under a compressive stress system with an isotropic component p and a deviatoric component q. Let this sample undergo a volumetric strain increment δv and shear strain increment $\delta\varepsilon$ due to stress increments δp and δq imposed on the sample. Then for a unit bulk volume of soil, the total energy supplied δE, the recoverable elastic energy stored δU and the irrecoverable plastic energy dissipated δW are related by:

$$\delta E = \delta U + \delta W = q\,\delta\varepsilon + p\,\delta v \quad .... (5)$$

Roscoe, Schofield and Thurairajah (1963) made the assumption that the change in internal energy is entirely due to the change in isotropic component δp in the applied stress and independent of the change in the deviatoric component δq. This is equivalent to assuming that the [illegible] all shear strain $\delta\varepsilon$ is irrecoverable. The recoverable energy from a unit bulk volume of clay which is in equilibrium at voids ratio e under a mean principal effective stress p, when p is increased by δp, has been shown to be $\delta U = \frac{\kappa\,\delta p}{1+e}$.

If $\frac{dW}{d\varepsilon}$ is denoted as q_w then

$$q_w = q + p\frac{dv}{d\varepsilon} - \frac{dU}{d\varepsilon} \quad ..... (6)$$

where $p\left(\frac{dv}{d\varepsilon}\right)$ and $\frac{dU}{d\varepsilon}$ are called the boundary energy correction and elastic energy correction respectively.

The results of triaxial compression tests on virgin consolidated kaolin presented in Fig. 3 and 4 shows that the corrected deviator stress q_w lies along the line $q_w = Mp$. It is evident from Fig. 3 and 4 that this result is valid for sand as well.

Hence the rate at which energy is dissipated during shear distortion of a unit bulk volume of soil which is in equilibrium under the mean principal effective stress p is assumed to be $\delta W = Mp\,\delta\varepsilon$.

The energy equation then becomes

$$q\,\delta\varepsilon + p\,\delta v - \frac{\kappa\,\delta p}{1+e} = Mp\,\delta\varepsilon \quad .... (7)$$

7.0 Volumetric Strain due to a Probing Stress Increment

Roscoe and Poorooshasb (1963) have

that deforms becomes stronger with deformation and hence the deformation spreads to other portions that are weaker. On the other hand, in a sample that "work-softens", the portion of the sample that deforms become weaker with deformation and hence the deformation continues in this zone. Therefore the results of triaxial compression tests on normally consolidated and lightly over-consolidated samples in which the samples "work-harden" during the shearing process have been used to develop the model for shear-deformation behaviour of soils.

3.0 Stress and Strain Parameters

The stress parameters used are the mean principal effective stress $p = 1/3(\sigma_1' + 2\sigma_3')$ and the deviator stress $q = (\sigma_1' - \sigma_3')$, where σ_1' and σ_3' are the effective axial and radial stresses respectively in the standard triaxial compression test. The strain parameters used are the natural volumetric strain v and the natural shear strain ϵ given by $\delta\epsilon = 2/3(\delta\epsilon_1 - \delta\epsilon_3) = (\delta\epsilon_1 - \frac{\delta v}{3})$, where $\delta\epsilon_1$ and $\delta\epsilon_3$ are the incremental natural axial strain and radial strain respectively. Compressive strains are considered to be positive.

4.0 Critical State Concept

A portion of the volume change that occurs when a sample of virgin consolidated clay is subjected to an increase in isotropic stress is recoverable on the removal of the stress. As shown in Fig. 1, in a plot of voids ratio e against $\log_e p$, where p is the isotropic stress, the virgin consolidation line NN is a straight line while the swelling line PQ obtained on unloading, though slightly curved, is assumed to be a straight line. Let the slope of the virgin consolidation line NN be $-\lambda$ and the slope of the swelling line PQ be - k.

When a sample of soil is sheared under undrained or drained conditions, the pore water pressure change or volume change that occur with further shear strain ceases at a certain stage. The soil is then said to be at a critical state. At this state, any further shear strain imposed on the soil will not produce any change in the stresses or voids ratio. As shown in Fig. 1, the points corresponding to the critical state lie on the line XX in $(e, \log_e p)$ plot, and this line is found to be parallel to the virgin consolidation line NN. In $(e, \log_e q)$ plot, the critical state points lie on the line XX shown whose slope is again $-\lambda$. In the (q,p) plot, the critical state points lie on a straight line that passes through the origin. Roscoe, Schofield and Wroth (1958) have shown the existence of the critical state line for cohesionless and cohesive materials.

The equation of the critical state line is given by,

$$q = Mp \qquad \text{........ (4)}$$
$$e = \Gamma - \lambda \log_e p$$

where Γ is the value of e on XX when p = 1.

5.0 Yield Surface

The isotrometric view of the yield surface for a clay, plotted in (p, q, e) space, is shown in Fig. 2. The domain that lies between the surface and the q = 0 plane is referred to as the yield domain. It is not possible for the state of the sample to lie outside this yield surface, and Roscoe and Poorooshasb (1963) referred to this surface as the state boundary surface. The loading paths for triaxial tests carried out on virgin consolidated clay lie on the curved part of the yield surface NNXX. Roscoe and Thurairajah

(1964) have shown the existence of a unique surface for any virgin consolidated clay on which all loading paths lie irrespective of the drainage conditions allowed.

ABCD in Fig. 2 represents the section of the state boundary surface by an e = constant plane. The curved line AD is the loading path followed by an undrained test on a sample of virgin consolidated clay. When the loading path reaches the point D, shear distortion takes place without change in stress or pore water pressure, and the state of the sample continues to remain at D. The portion of the surface XXMM is a ruled surface. CD is a straight line and the intercept BC on the p = 0 plane increases with decrease in e. The loading path for a heavily over-consolidated sample of clay rises from the q = 0 plane, lying initially inside the state boundary surface, and reaches the ruled part of the surface. The path thereafter moves on this surface and ends on the line XX at the critical state.

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Consider a triaxial sample of soil which is in equilibrium under a compressive stress system with an isotropic component p and a deviatoric component q. Let this sample undergo a volumetric strain increment δv and shear strain increment $\delta \epsilon$, due to stress increments δp and δq imposed on the sample. Then for a unit bulk volume of soil, the total energy supplied $\delta E'$, the recoverable elastic energy stored δU and the irrecoverable plastic energy dissipated δW are related by :

$$\delta E' = \delta U + \delta W = q\,\delta E + p\,\delta v \quad \ldots. \quad (5)$$

Roscoe, Schofield and Thurairajah (1963) made the assumption that the change in internal energy is entirely due to the change in isotropic component δp of the applied stress and is independent of the change in the deviatoric component δq. This is equivalent to assuming that the soil is rigid plastic in shear distortion i.e. all shear strain $\delta \epsilon$ is irrecoverable. The recoverable energy from a unit bulk volume of clay which is in equilibrium at voids ratio e under a mean principal effective stress p, when p is increased by δp , has been shown to be

$$\delta U = \frac{k\,\delta p}{1+e}$$

If $\frac{dW}{dE}$ is defined as q_w, then

$$q_w = q + p\,\frac{dV}{dE} - \frac{dU}{dE} \quad \ldots\ldots \quad (6)$$

where $p\left(\frac{dV}{dE}\right)$ and $-\frac{dU}{dE}$ are called the boundary energy correction and elastic energy correction respectively.

The results of triaxial compression tests on virgin consolidated kaolin presented in Fig. 3 and 4 shows that the corrected deviator stress q_w lies along the line $q_w = Mp$. It is evident from Fig. 5 and 6 the this result is valid for sand as well.

Hence the rate at which energy is dissipated during shear distortion of a unit bulk volume of soil which is in equilibrium under the mean principal effective stress p is assumed to be $\delta W = Mp\,\delta E$.

The energy equation then becomes,

$$q\,\delta E + p\,\delta v = \frac{k\,\delta p}{1+e} + Mp\,\delta E \quad \ldots. \quad (7)$$

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Roscoe and Poorooshasb (1963) have

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The stress parameters used are the mean principal effective stress $p = 1/3(\sigma_1 + 2\sigma_3)$ and the deviator stress $q = (\sigma_1 - \sigma_3)$, where σ_1 and σ_3 are the effective axial and radial stresses respectively in the standard triaxial compression test. The strain parameters used are the natural volumetric strain v and the natural shear strain ϵ given by $\delta v = \delta\epsilon_1 + 2\delta\epsilon_3$, $\delta\epsilon = \frac{2}{3}(\delta\epsilon_1 - \delta\epsilon_3)$ where $\delta\epsilon_1$ and $\delta\epsilon_3$ are the incremental natural axial strain and radial strain respectively. Compressive strains are considered to be positive.

The overall volumetric strain increment can be separated into two components

$$\delta v = \delta v^e + \delta v^p \quad ......$$

where δv^e and δv^p represent the elastic and plastic components respectively. The isotropic swelling line is used to separate these two components. The plastic component is therefore given by,

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When a sample of soil is sheared under undrained or drained conditions, the pore water pressure change or volume change that occur with further shear strain ceases at a certain stage. The soil is then said to be at a critical state. At this state any further shear strain imposed on the soil will not produce any change in the stresses or voids ratio. As shown in Fig. 1, the points corresponding to the critical state lie on the line XX in (e, log p) plot and this line is found to be parallel to the virgin consolidation line NN. In (e, $\log_e$ q) plot, the critical state points lie on the line XX shown whose slope is again $-\lambda$. In the (q,p) plot, the critical state points lie on a straight line that passes through the origin. Roscoe, Schofield and Wroth (1958) have shown the existence of the critical state line for cohesionless and cohesive materials.

The equation of the critical state line is given by,

As stated earlier, it has been assumed that $q = Mp$ and $\delta\epsilon^e = 0$ and $\delta\epsilon = \delta\epsilon^p$. .... (4)

For the critical state line $e = \Gamma - \lambda \log_e p$, where Γ is the value of e on XX when p = 1.

5.0 Yield Surface

The isometric view of the yield surface for a clay, plotted in (p, q, e) space, is shown in Fig. 2. The domain that lies between the surface and the q = 0 plane is referred to as the yield domain. It is not possible for the state of the sample to lie outside this yield surface. Roscoe and Poorooshasb (1963) referred to this surface as the state boundary surface. The loading paths for triaxial tests carried out on virgin consolidated clay lie on the curved part of the yield surface NNXX. Roscoe and Thurairajah

shown that the loading paths of undrained tests carried out on virgin consolidated clay, plotted on (p, q) space, are geometrically similar. Let $S = -(dq/dp)$ represent the slope of the loading path of an undrained test for any value of $\eta = q/p$. S is a unique function of η. Hence the state boundary surface can be defined geometrically in terms of the slope of the curves formed by the intersection of this surface by a family of e constant planes.

Consider an element of clay with state (e, p, q) lying on the state boundary surface. If a probing stress increment (δp, δq) is applied to the element such that the stress path remains on the state boundary surface, the change in voids ratio δe is determined by the shape of the state boundary surface and is given by:

$$\delta e = -\frac{\lambda}{p}\frac{(\delta q + S\,\delta p)}{S + \eta} \quad .... (8)$$

The corresponding volumetric strain increment is given by

$$\delta v = \frac{-\delta e}{1+e} = \frac{\lambda(\delta q + S\,\delta p)}{p(1+e)(S+\eta)} \quad .... (9)$$

$$\delta v^p = \frac{\lambda - k}{1+e}\frac{\delta p}{p} \quad .... (11)$$

8.0 Plastic Potential

Calladine (1963) suggested that if the curve on the state boundary surface CHF in Fig. 7, lying vertically above the elastic swelling line PQ is projected on to the (p, q) plane, the curve obtained represents the yield locus satisfying the normality condition postulated by Drucker (1951) for the non-linear elastic material formed by isotropic virgin consolidation and subsequent unloading. The curved vertical wall CFE represents all elastic states which can be reached from CQ without yielding. The curve CHF represents the elastic limit and the projection of CHF on to the e = 0 plane forms the yield locus of the plastic potential.

9.0 Shear Strain due to a Probing Stress Increment

For a state path lying on the state boundary surface, if the plastic strain increment vector is normal to the yield locus, then

$$\left(\frac{dq}{dp}\right) = -\frac{dv^p}{d\epsilon} \quad ...... (12)$$

For the elastic limit curve CF, since $\delta e = -k\frac{\delta p}{p}$, from Eq. (8),

$$\left(\frac{dq}{dp}\right)_{CF} = -\frac{k}{\lambda}\left[S\left(\frac{\lambda}{k} - 1\right) - \eta\right] \quad ...(13)$$

From Eq. (8), (11), (12) and (13).

$$\delta\epsilon = \frac{\lambda}{p(1+e)}\left[\frac{\frac{\lambda}{k}\delta q + \{S(\frac{\lambda}{k}-1) - \eta\}\delta p}{(S+\eta)\{S(\frac{\lambda}{k}-1) - \eta\}}\right] \quad (14)$$

Alternately if the energy equation is applied to a state path lying on the state boundary surface, (1960) from Eq. (7) and (9).

$$\delta\epsilon = \frac{\frac{\lambda}{k}\delta q + \{S(\frac{\lambda}{k}-1) - \eta\}\delta p}{p(1+e)(S+\eta)(M-\eta)} \quad (15)$$

10.0 Cam Clay Model

For a material that satisfies both the energy equation and the normality condition, the yield locus, state boundary surface and stress-strain relationship can be predicted. This material is called Cam clay and the equations derived are given below ;

i) Stress-path for undrained tests;

$$S = \frac{M}{(1-\frac{k}{\lambda})} - \eta \quad \ldots\ldots (16)$$

$$q = \frac{M}{(1-\frac{k}{\lambda})} p \log_e(p_o/p) \quad \ldots(17)$$

where $p = p_o$ when $q = \eta = 0$

ii) State boundary surface;

$$q = \frac{M}{(\lambda-k)} p \quad (e_a - e - \lambda \log_e p) \quad \ldots(18)$$

where e_a is the value of e on the isotropic virgin consolidation line when p = 1.

iii) Yield Locus;

$$q = Mp \log_e (p_o/p) \quad \ldots. (19)$$

iv) Stress-strain relationship;

$$\delta e = - \frac{\lambda-k}{Mp}(\delta q - \eta \delta p) + \frac{\lambda}{p}\delta p \quad \ldots. (20)$$

$$\delta \epsilon = \frac{\lambda-k}{Mp-(1+e)} \left[\frac{\delta q}{M-\eta} + \delta p\right] \quad \ldots. (21)$$

It should be noted that using this simple model the strains can be predicted making use of the fundamental constants λ, k, M and e_q', all of which can be determined from simple tests.

Hata, Ohta and Yoshitani (1969) carried out triaxial compression tests on an alluvial clay taking great care in preparing the samples. The loading path for a virgin consolidated sample sheared under undrained conditions is presented in Fig. 8. These results confirm the assumption made in this model that the yield surface is bullet shaped.

The yield locus for Cam-clay is shown in Fig. 9. It is assumed that the yield locus is symmetrical about the p-axis. OX is the projection of the critical state line. The equation for the yield locus shows that at C where the critical state line OX intersects the yield locus, the tangent to the yield locus is parallel to the p-axis and the value of p is $p_o/2.72$. Hence the plastic strain increment vector at C will be parallel to the q-axis confirming that no plastic volumetric strain is possible at the critical state C. The plastic strain increment vectors to the left of C indicates that shear distortion in this state will produce expansion in volume of the sample while that to the right C indicates that shear destortion will produce contraction in the sample.

11.0 Derivation of Cam Clay Equations

The Cam clay is a material that satisfies the following assumptions;

i) The isotropic virgin consolidation line has the equation

$$e = \Gamma + \lambda - k - \lambda \log_e p = e_a - \lambda \log_e p \quad \ldots\ldots(22)$$

and the isotropic swelling and recompression line has the equation

$$e = e_b - k \log_e p \quad \ldots\ldots \quad (23)$$

where Γ, λ , k, e_a are soil constants.

ii) Elastic shear strains are zero.

iii) Elastic volumetric strains are given by the isotropic swelling and recompression line.

iv) During yielding, energy dissipated per unit bulk volume of soil is $Mp\,\delta E^p$. Hence

$$q\,\delta E^p + p\,\delta v^p = Mp\,\delta E^p \quad .... (24)$$

This equation represents the yield function in theory of plasticity.

v) Drucker's normality condition is applicable, i.e. the plastic strain increment vectors are normal to the yield locus.

vi) The size of the yield locus is fixed by the value of the pressure at the intersection of the isotropic virgin consolidation line and the corresponding swelling line. This gives the hardening law in theory of plasticity.

The equations given in the earlier sections for the Cam clay can be derived from these assumptions. It should be noted that the equation of the critical state line is not necessary to derive these equations, and the critical state concept about the behaviour of soil is a consequence of the above assumptions from theory of plasticity.

12.0 Modified Cam Clay

The yield function and the hardening law used for deriving Cam clay equations can be modified to produce slightly different soil models. Modified Cam clay model presented by Roscoe and Burland (1968) meets some of the deficiencies of the Cam clay model. In this model, the energy dissipated per unit bulk volume of the soil is taken as

Hence,

$$q\,\delta E^p + p\,\delta v^p = p\sqrt{(\delta v^p)^2 + (M\,\delta E^p)^2} \quad .... (25)$$

The yield function for the model is

$$\frac{\delta v^p}{\delta \epsilon^p} = \frac{M^2 - \eta^2}{2\eta} \quad ...... (26)$$

The yield locus for modified Cam clay then becomes,

$$q^2 + M^2 p^2 = M^2 p\, p_o \quad .... (27)$$

which is elliptical in shape, as shown in Fig. 10.

The Cam clay model assumes that the strains are entirely elastic when the loading path lies below the state boundary surface. Experiments show that the concept of an elastic limit line lying on the state boundary surface is a good approximation for volumetric strains. However, plastic shear strains are found to take place when the loading paths lie below the state boundary surface. The revised yield locus suggested by Roscoe and Burland (1968) can be used to estimate these strains.

13.0 Concluding Remarks

Coulomb's theory was the framework for soil mechanics for about 150 years, and there was nothing much before that. Then came Terzaghi's theory of effective stress which led to separate analysis of shear strength and consolidation. Since then, critical state soil mechanics theory has made a revolutionary change in the discipline of soil mechanics. Critical state soil mechanics is the only current theory which deals at the same time with strength, stress-strain behaviour and consolidation of normally consolidated and over-consolidated soils. It gives not only a simple qualitative view of soil behaviour but also a complete mathematical model for soils.

Cam clay model can be used to describe the behaviour of soil under different combination of stresses in terms of some fundamental constants of the soil which can be determined from simple tests. It presents a simplified picture of the behaviour of soil. The equations derived from the model can be used to predict the deformation in the soil mass under different stress conditions.

Many deficiencies have been pointed out by research workers on the Cam clay model which was proposed in 1963, and modifications to the model have been suggested. The model has been successfully used in predicting the performance in soil engineering problems involving soft clays and lightly over-consolidated clays e.g. embankments and oil tanks on soft ground.

It should be noted that attempts made by research workers to improve theoretical prediction of soil behaviour make use of the concepts of critical state soil mechanics. Mroz and Norris (1982) proposed a soil model with smaller yield loci nested inside a larger yield locus. Dafalias and Herrmann (1982) proposed a bounding surface model where the amount of plastic deformation associated with a stress point inside the bounding surface depends on the distance to an image point on the surface. Other models have been suggested by Pender (1982) and Naylor (1985). These models are more complicated than Cam clay and their application involves laborious calculations for prediction of behaviour.

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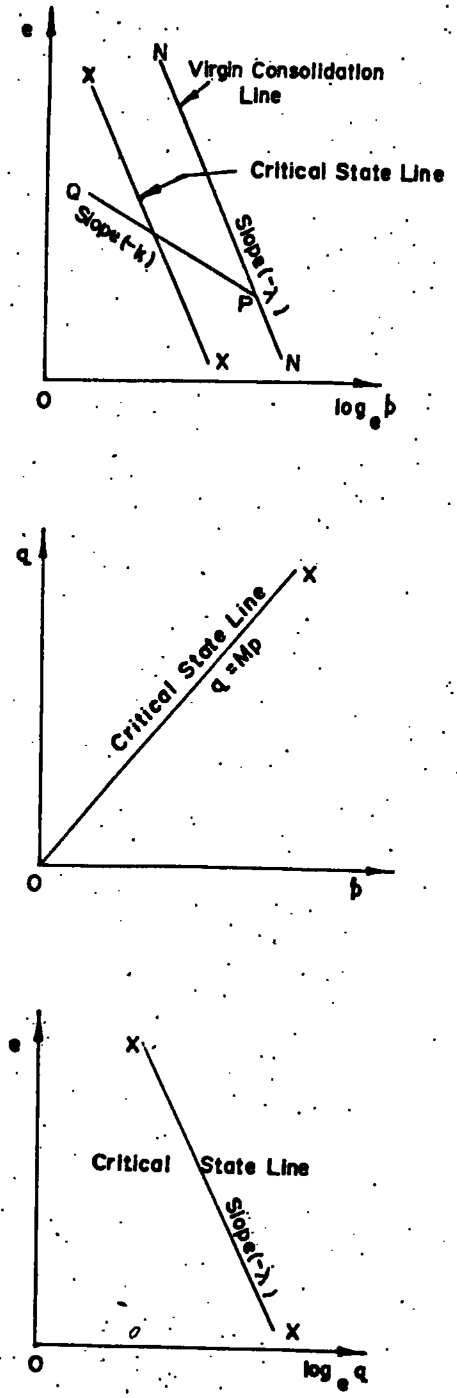
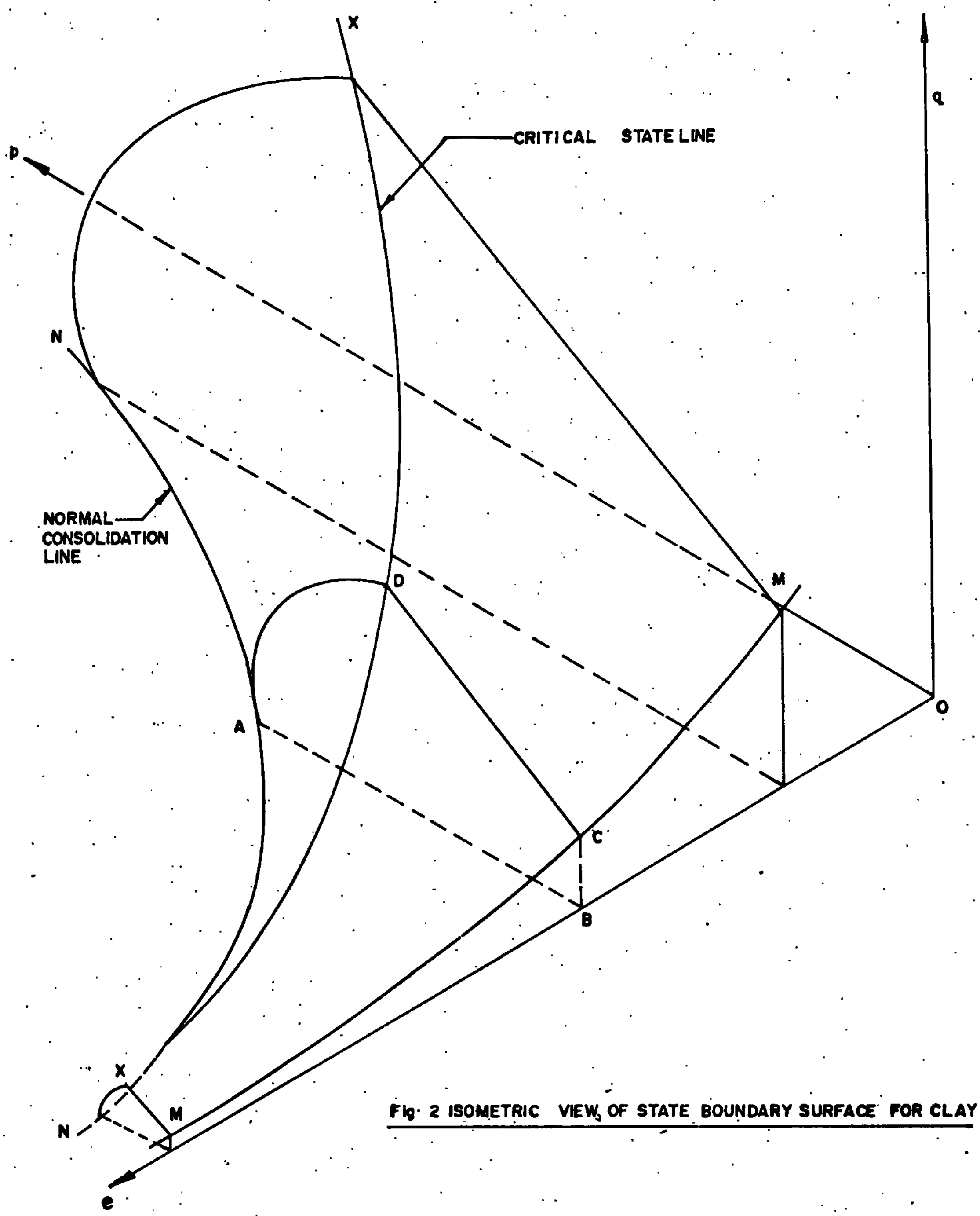
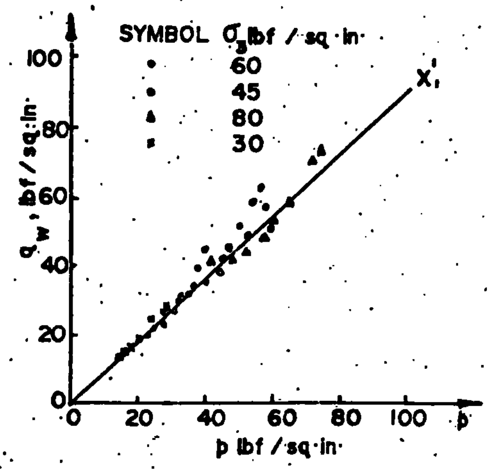


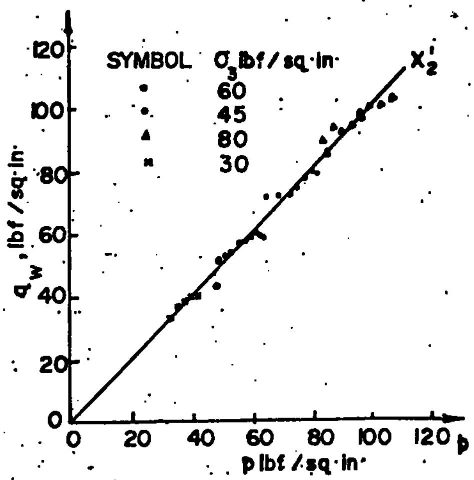
Fig · 1 PROJECTIONS OF CRITICAL STATE LINE



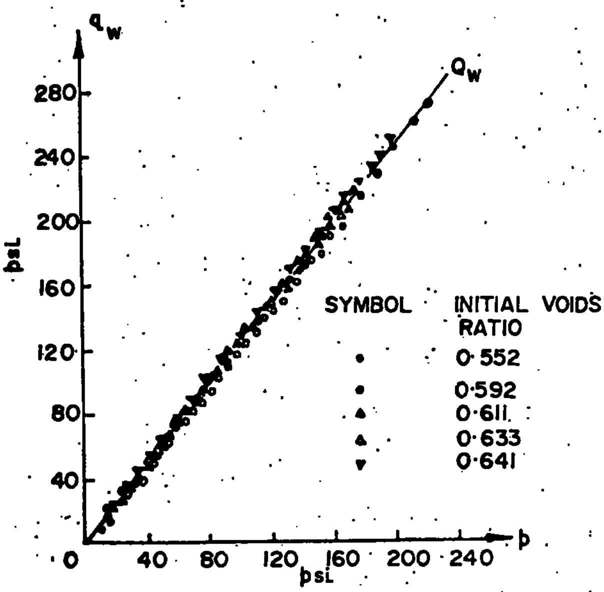
Fig· 2 ISOMETRIC VIEW OF STATE BOUNDARY SURFACE FOR CLAY



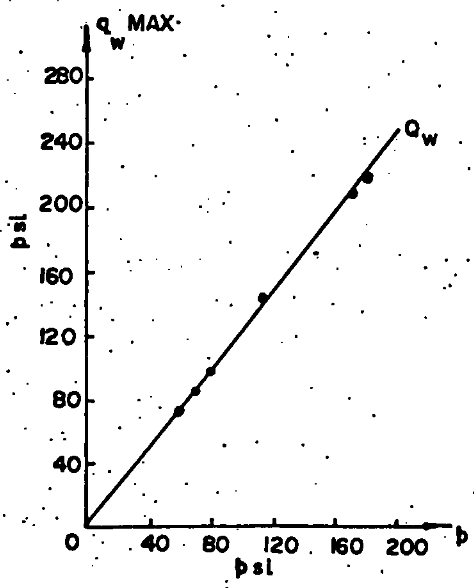
Fig·3 VARIATION OF q_w WITH p. FOR UNDRAINED TRIAXIAL COMPRESSION TESTS ON ISOTROPIC VIRGIN CONSOLIDATED KAOLIN·



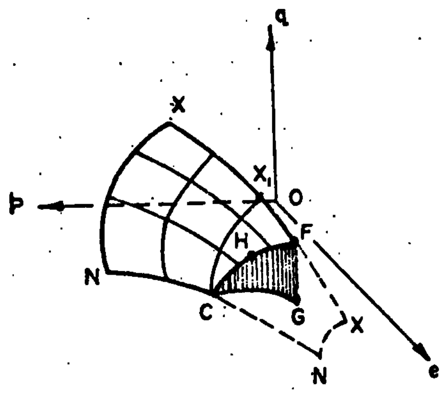
Fig·4 VARIATION OF q_w WITH p FOR DRAINED TRIAXIAL COMPRESSION TESTS ON ISOTROPIC VIRGIN CONSOLIDATED KAOLIN·



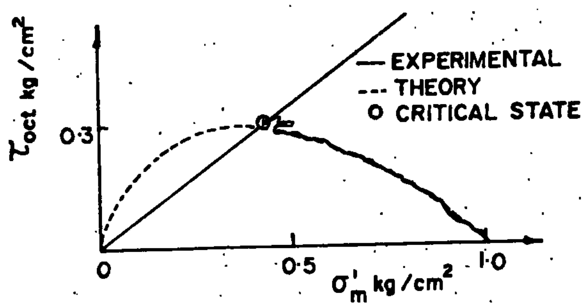
Fig·5 VARIATION OF q_w WITH p DURING UNDRAINED TRIAXIAL COMPRESSION TESTS ON SAND·



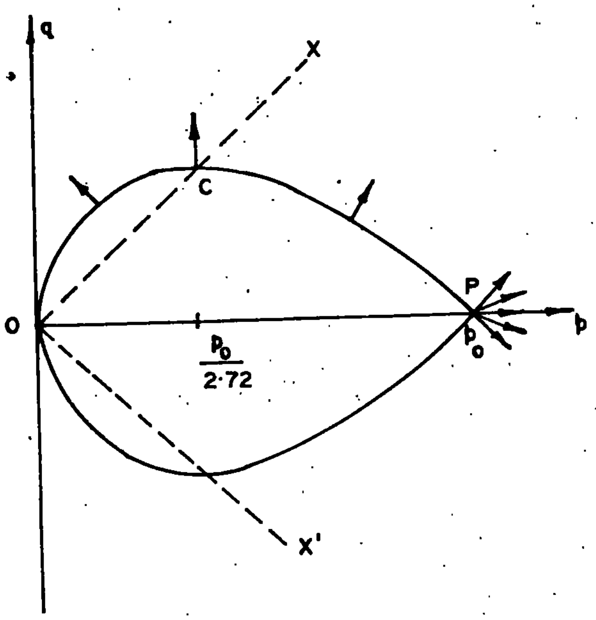
Fig·6 PEAK POINTS OF q_w DURING DRAINED TRIAXIAL COMPRESSION TESTS ON SAND. THE LINE OQ_w IS TAKEN FROM Fig·5



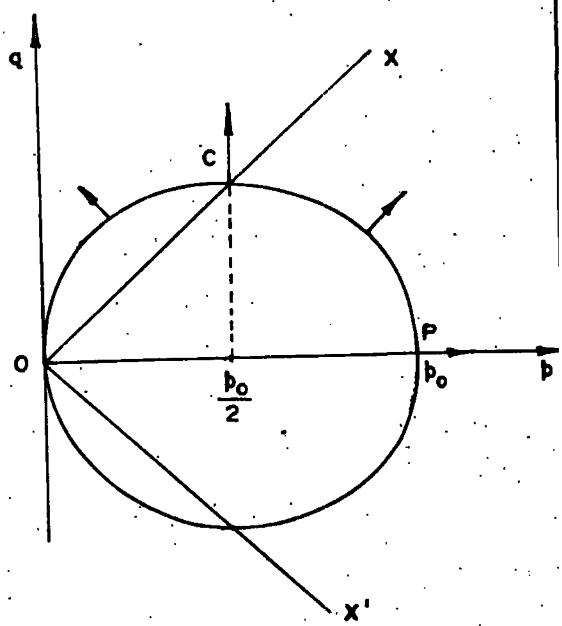
Fig·7· THE ELASTIC SURFACE CFG AND ELASTIC LIMIT CURVE CHF·



Fig·8 STRESS PATH OF NORMALLY CONSOLIDATED CLAY UNDER UNDRAINED SHEAR ·



Fig·9· YIELD LOCUS FOR CAM-CLAY



Fig·10· YIELD LOCUS FOR MODIFIED CAM · CLAY