NUMERICAL MODELLING OF TIDAL CURRENTS IN THE NEGOMBO LAGOON, SRI LANKA

by

Prof. P. Holmes[1], and S.P.Samarawickrama[2]

Abstract

The paper describes the application of a numerical model to predict water elevations, the two dimensional tide induced velocity field and the residual currents in the Negombo Lagoon, Sri Lanka. The water elevations and the depth averaged velocities within the lagoon were evaluated numerically by solving the time dependent non-linear equations of mass and momentum, in the horizontal plane, using the alternating direct implicit finite difference scheme. The two depth integrated momentum equations were formulated to include the effects of the earth's rotation, the bed resistance, turbulent diffusion and convective acceleration. Only few field measured velocity variations and water elevations were available to compare with mathematically predicted values. The resulting comparisons have shown an encouraging agreement, particularly in view of the coarseness of the computational grid spacing, and consequently the coarse representation of the bathymetry, and the lack of appropriate bed resistance data.

1.0 Introduction

The Negombo lagoon is a shallow coastal body of water located on the west coast of Sri Lanka ($7^0 10^1$ N and $79^0 50^1$ E). The lagoon is approximately 12.5km in length and its width varies from 0.6 to 3.6 km, Figure 1. Its mean depth is estimated to be approximately 0.65m and the surface area to be 35km^2, thus placing its volume to be of the order of 22.5 million m^3. One of the unique features of the lagoon is that its transition to the sea consists of several narrow channels. The total cross-sectional area of the inlet channels is estimated to be 250m^2 with of length of about 2.5km.

The exchange of water in the lagoon is influenced by the tides from the ocean and fresh water supply from inland. The tide is semi-diurnal and the tidal range in the lagoon varies in the order of 0.07m at neaps to 0.2m at springs, these values being about one third of the tide at sea. Thus the volume of water stored and released varies between 2.5 million m^3 and 7.0 million m^3 per tide. Fresh water enters from the southern end of the lagoon through several small streams. The supply of fresh water varies from virtually zero during the dry seasons to more than 100 m3/s during the rainy seasons.

In recent years there has been a growing concern about the water-quality problems associated with tidal flushing and circulation patterns in the Negombo lagoon, particularly in the planning and design stage of modifications etc. Excessive siltation has taken place in the lagoon resulting in the loss of about 25% of the water area during the last three decades. This has decreased the tidal flow in and out of the lagoon, thus reducing the current velocities during flood and ebb tides which are required to keep the inlet open with sufficient depth for navigation. The reduction of the tidal flow could in the long term adversely affect the lagoon environment. A depth averaged, finite difference numerical model was developed and used to predict water surface elevation, velocity fields and residual movements in the Negombo lagoon. The conservation equations of mass and momentum were solved using the well documented Alternating Direct Implicit (ADI), Finite Difference technique which was second order accurate in space and time. In the momentum equations the effects of the earth's rotation, bed friction, turbulent diffusion and convective accelerations were all included. The numerical solution procedure involved the establishment of a regular mesh of square grids over the area of interest, with the conservation equations being expressed in an implicit finite difference form. The resulting simultaneous equations, together with the known hydrodynamic conditions at the open and closed boundaries, were then solved by the method of Gauss elimination and back substitution to give the water surface elevations and the two mutually perpendicular depth averaged velocity components in the horizontal plane for each "wet" square.

Prof. P Holmes, Professor -Head of Hydraulics Section, Imperial College of Science, Technology and Medicine, UK.

Mr. S P Samarawickrema, Research Student, Imperial College of Science Technology and Medicine, UK.

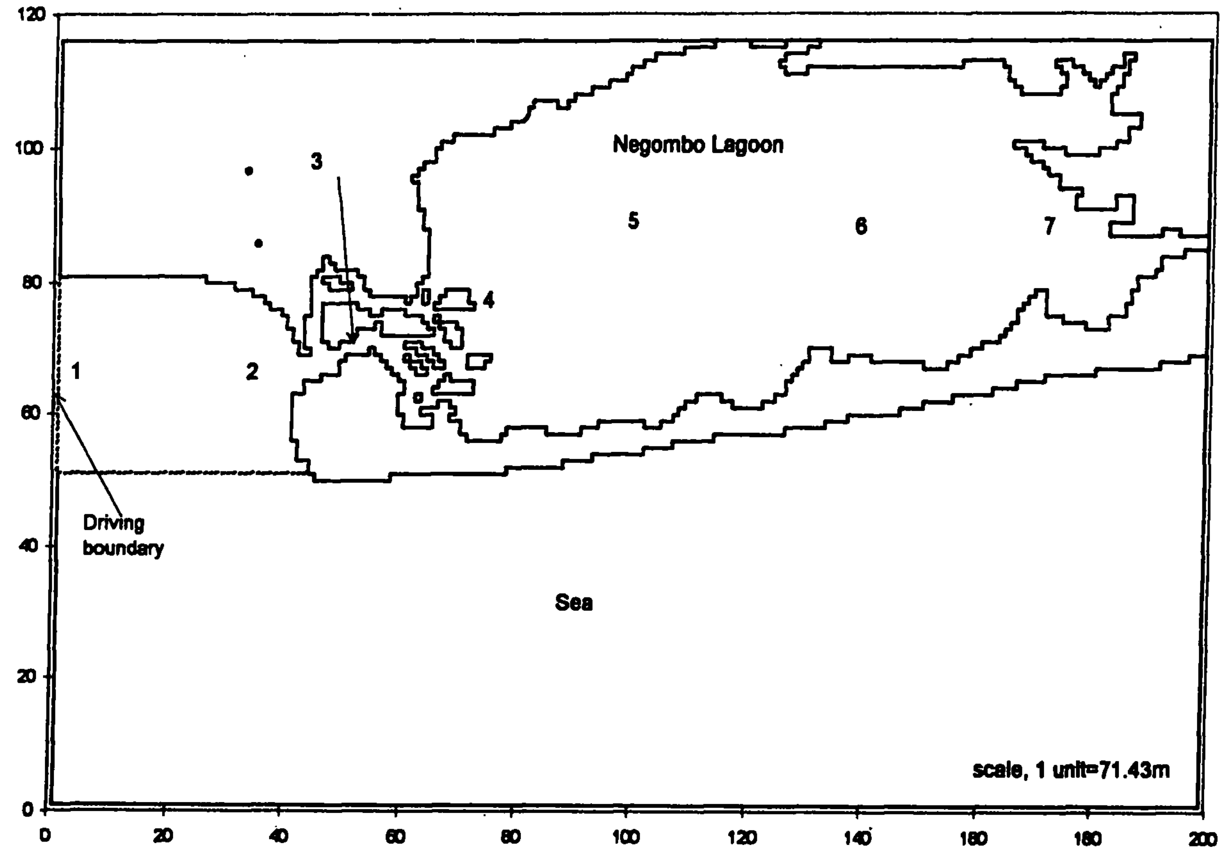

Figure 1. Plan view of the Negombo Lagoon showing the driving boundary and output points

2. 0 Governing Equations

The depth averaged two-dimensional navier-Stokes equations which describe tidal motion are as follows:

$$\frac{\partial UH}{\partial t}+\frac{\partial U^2H}{\partial x}+\frac{\partial UVH}{\partial y}-fVH+gH\frac{\partial \eta}{\partial x}+\frac{\xi_{bx}}{p}-\vartheta_t H\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right)=0 \quad (1)$$

$$\frac{\partial VH}{\partial t}+\frac{\partial UVH}{\partial x}+\frac{\partial V^2H}{\partial y}-fVH+gH\frac{\partial \eta}{\partial x}+\frac{\xi_{by}}{p}-\vartheta_t H\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right)=0 \quad (2)$$

$$\frac{\partial \eta}{\partial t}+\frac{\partial UH}{\partial x}+\frac{\partial VH}{\partial y}=0 \quad (3)$$

$$H=d+\eta \quad (4)$$

where,

U	is the depth averaged velocity component in the x direction,
V	is the depth averaged velocity component in the y direction,
H	is the total water depth,
d	is the mean water level (datum),
η	is the water surface displacement from the datum,
f	is the Coriolis parameter,
ξ_{bx}	is the bed shear stress component in the x direction,
ξ_{by}	is the bed shear stress component in the y direction
ϑ_t	is the depth averaged eddy viscosity,
g	is the gravitational acceleration,
p	is the density of water,
t	is time.

Equations 1,2 and 3 are the x-momentum, y momentum and conservation of mass equations respectively.

The mean eddy viscosity was defined, Falconer (1984), by

$$V_t = \frac{\beta \sqrt{g}\,(h+\eta)\,\sqrt{U^2+V^2}}{C}$$

where,

$\beta = k/6$

k - von karmans constant

$C = 1\,(h+\eta)^{1/6}$

n- Mannings Coefficient

3.0 Solution Grid And Boundary Conditions

A staggered grid was used for model calculations. For each cell the associated η value corresponds to its center and associated velocities U and V at the mid-points of the right and top sides of the cell, respectively, were calculated. The U and V values of the other two sides were considered as associated with the adjacent cells. The water depths measured from the datum were known at the grid cell corners.

The region of the Nogombo lagoon simulated in the mathematical model was defined by a finite difference mesh of 199 by 66 grid points, with an equal grid spacing of 71.43m in both directions. As in the case with all numerical models, the placement of the boundaries and the conditions there greatly influence the solution process.

In modeling the tidal currents in the Negombo Lagoon, it was necessary to introduce a time-varying water surface elevation at the open seaward boundary, Figure 1-driving boundary. The velocity parallel to this driving boundary was set equal to zero, as were the velocities along the non-driving boundaries. The velocity components at right angles to the non-driving boundaries were also set equal to zero.

For the seaward boundary previously measured water elevations were available, (Hettiarachchi,1995) and the average Tidal Range of 0.6m was used for this study. Depth contours of the Negombo Lagoon were available, (Terwel & Vermeulen,1995) and water depths of each and every grid point were obtained using those contours. No appropriate field measurements were available for the bed roughness characteristics along the lagoon reach. The model was tested for various values of Manning's n, and n=0.03 was found to be most appropriate for this study. A small time step of 69.94 seconds was used to overcome numerical instability problems, which might have arisen because of the complicated geometry of the application.

4.0 Discussion of Results

The model was run on all occasions for three complete tidal cycles, with the results of the first half cycle being disregarded since the model needed some time to dissipate the errors introduced at the commencement of each simulation. Seven important locations within the computational domain were selected to represent the outputs of the variation of velocity and water depth with time, Figure 1. Water elevation above the datum and the depth averaged velocity components in the horizontal plane were evaluated for all of those seven points, Figures 2 & 4. Also graphical outputs of the spatial velocity fields were produced during the second tidal cycle at mean water level on flood, Figures 5 & 6, three-quarter flood tide, Figure 7, three-quarter ebb tide, Figure 8 and mean water level on ebb tide, Figure 9. Figure 10 shows the residual movements, the net displacement of water over a complete tidal cycle, of some selected points.

Only few field measured velocity variations and water elevations were available to compare with mathematically predicted values. Variation of water level at a point inside the lagoon, Figure 3 and another near the coast were available for a four day period. A measurement near the coast was used to get the average tidal range, which was used as the model input data. Measured water surface elevation at a point inside the lagoon with the corresponding external tidal fluctuations are shown in Figure 3. the predicted water level variation at the corresponding location within the lagoon are also shown in the same figure. The magnitude of predicted level changes shows good agreement with the full-scale data. The longer term variations in the mean water level seen in Figure 3 both outside and inside the lagoon are thought to be due to surge motion which merit further consideration. The agreement between the mathematically predicted and the field measured results is considered encouraging, particularly in view of the coarseness of the grid and the consequent difficulty of representing such a complex bathymetry at discrete points.

Difficulty in assessing bed roughness characteristics without appropriate field data, the exclusion of wind speeds from the mathematical model and the use of sinusoidal water level variation as the input instead of actual semi-diurnal tidal variation have contributed to the difference between measured and predicted values. Some discrete velocity measurements were available at the lagoon mouth (point 3, fig.1), with a

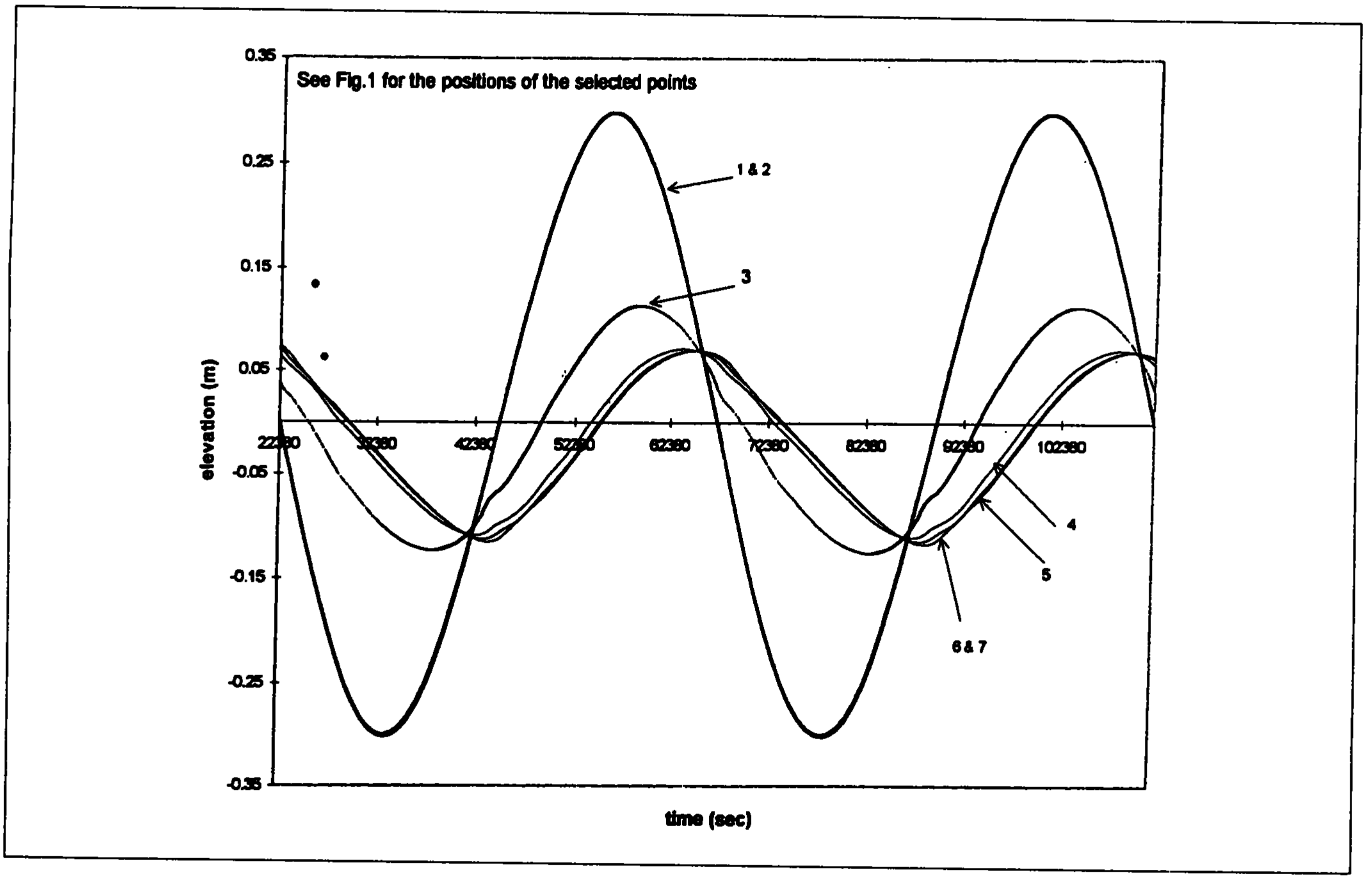

Figure 2. Predicted Water-Surface Elevations vs Time (Sec)

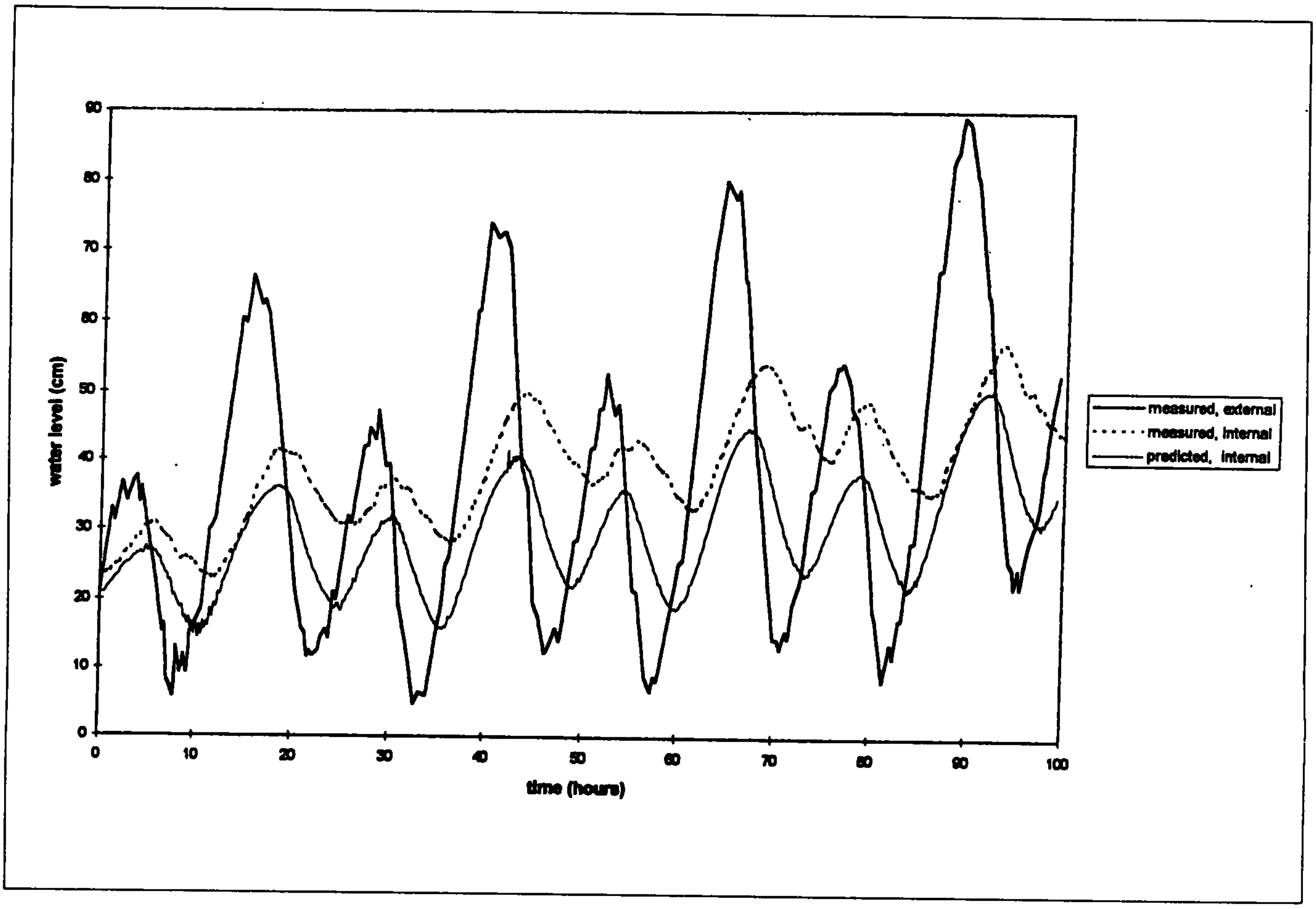

Figure 3. Measured and Predicted Water Depths vs Time (hours)

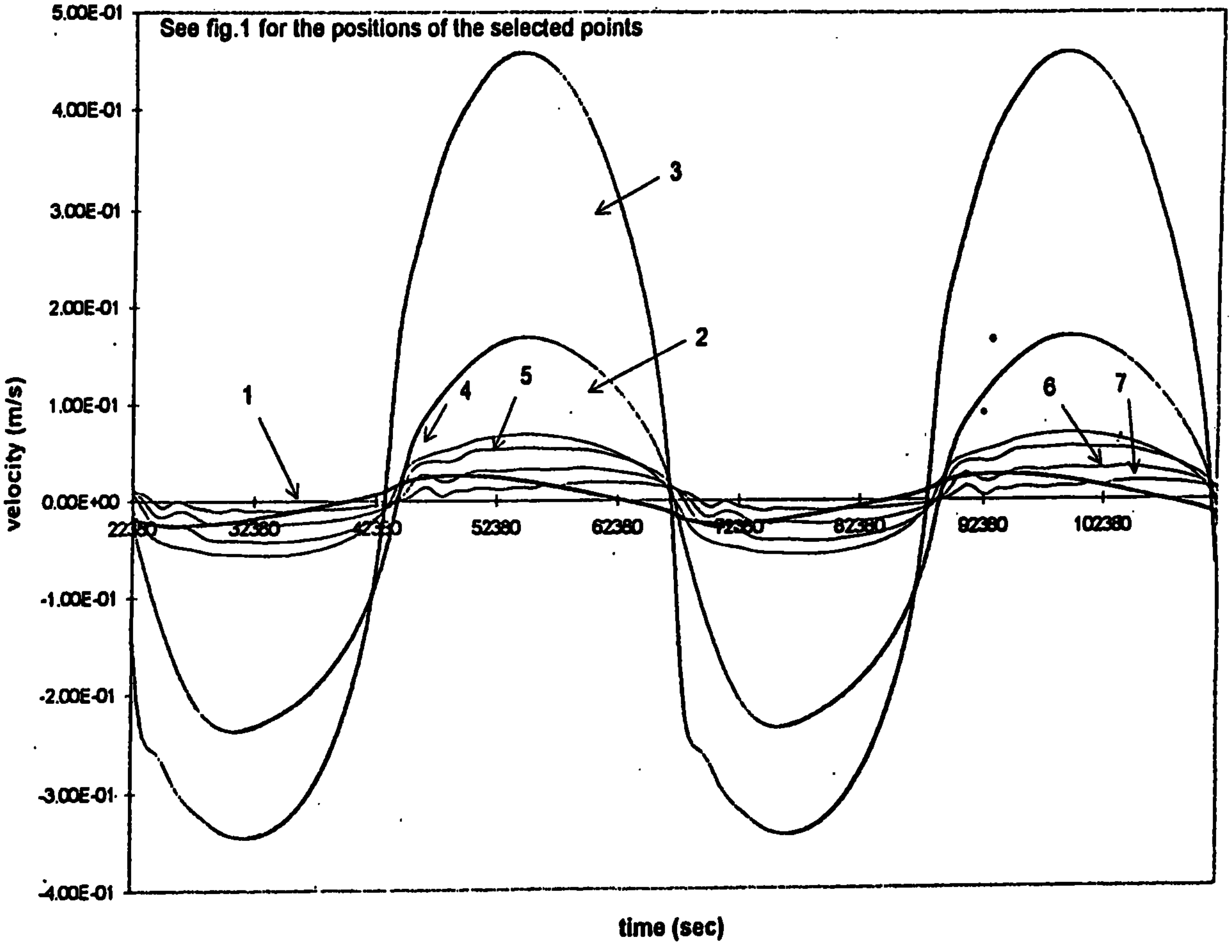

Figure 4. Velocity Magnitude vs Time (Sec)

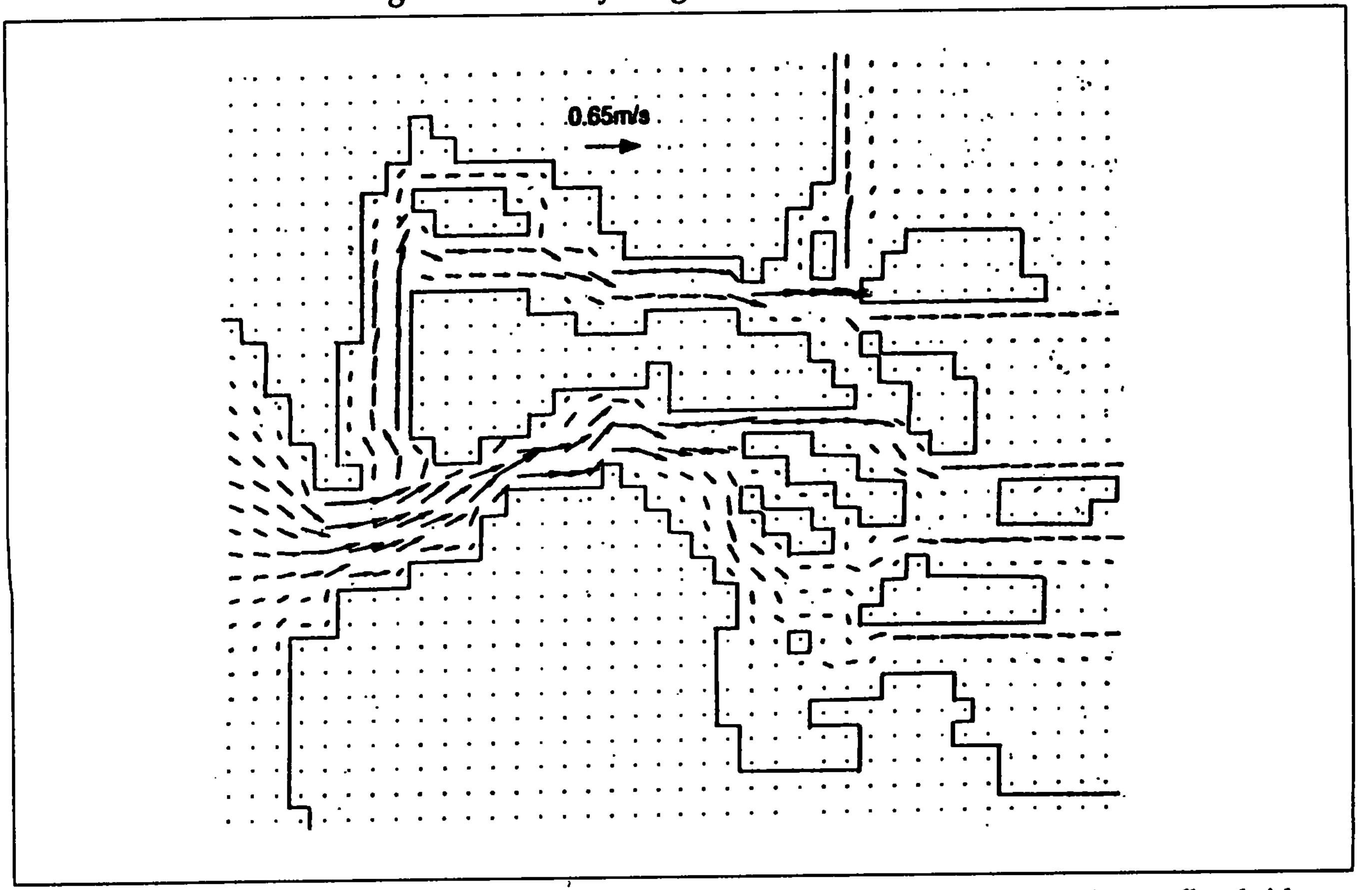

Figure 5. Typical velocity vector plot at the Lagoon mouth for mean water level on a flood tide.
(Vector scale given by separate arrows)

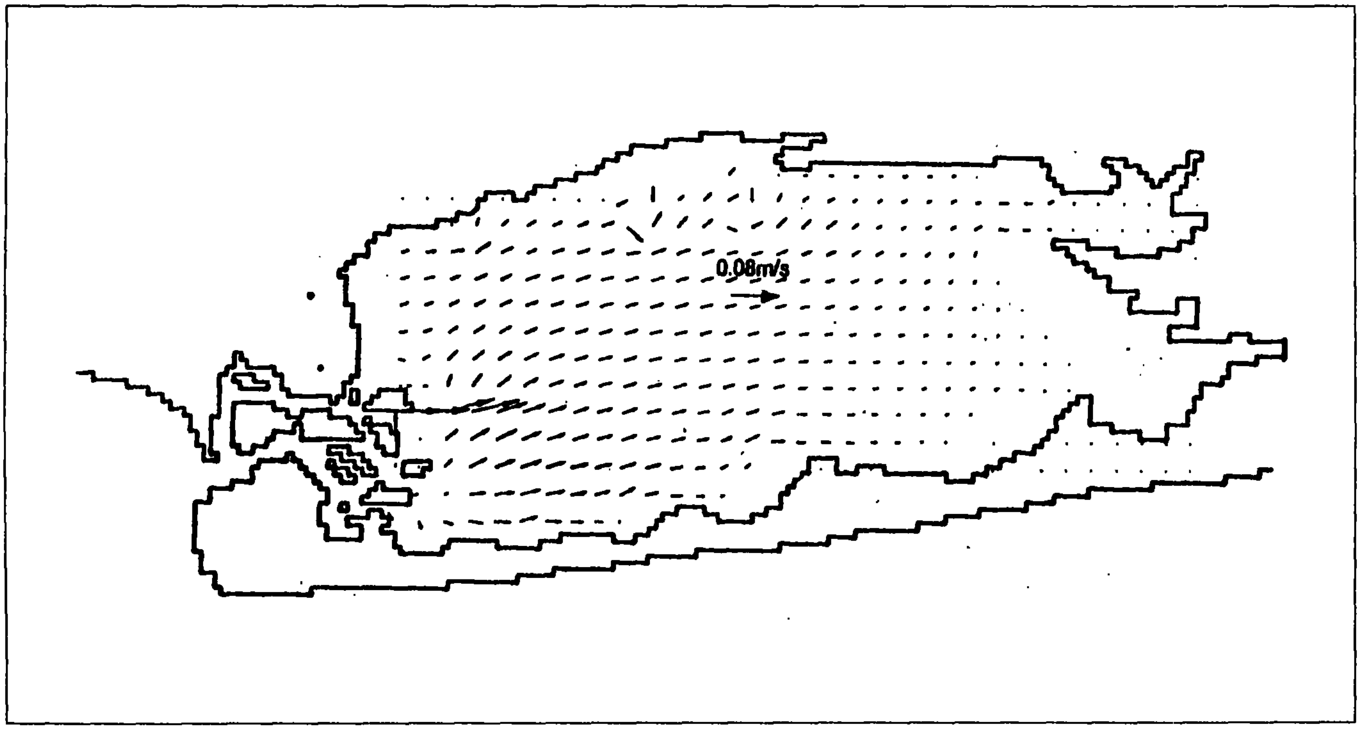

Figure 6. Velocity vector plot in the Lagoon at mean water level on a flood time.
(Vector scale given by separate arrows)

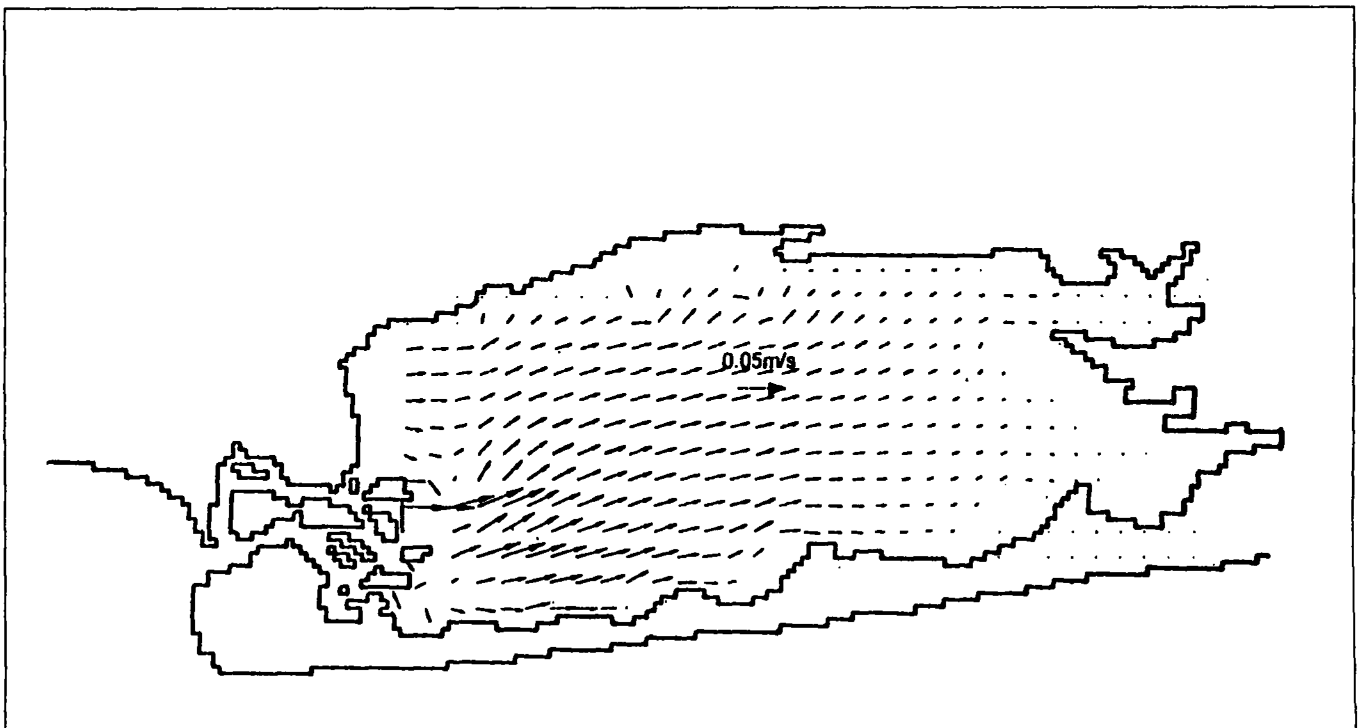

Figure 7. Velocity vector plot in the Lagoon at three-quarter flood tide
(vector scale given by separate arrows)

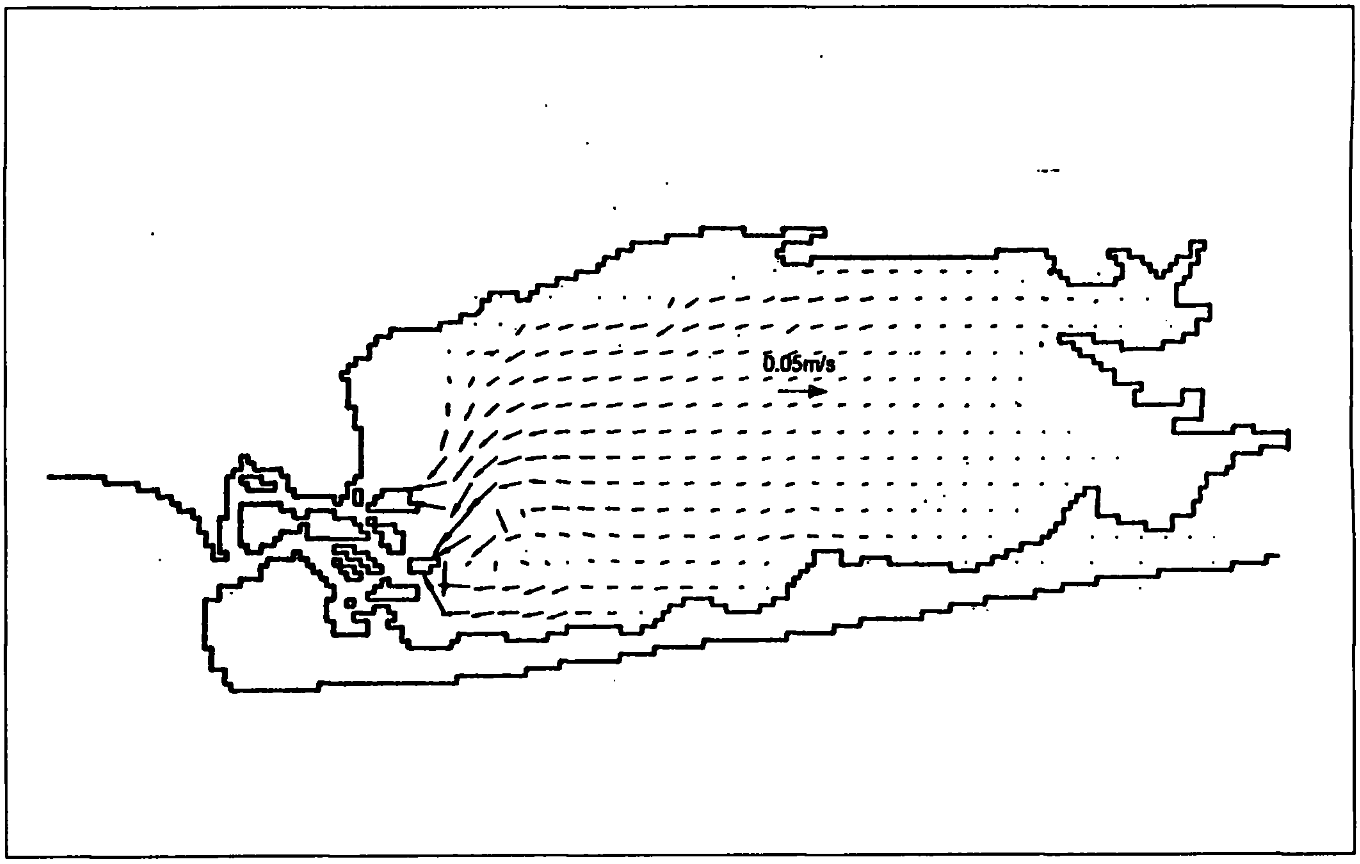

Figure 8. Velocity vector plot in the Lagoon at three-quarter ebb tide.
(Vector scale given by seperate arrows)

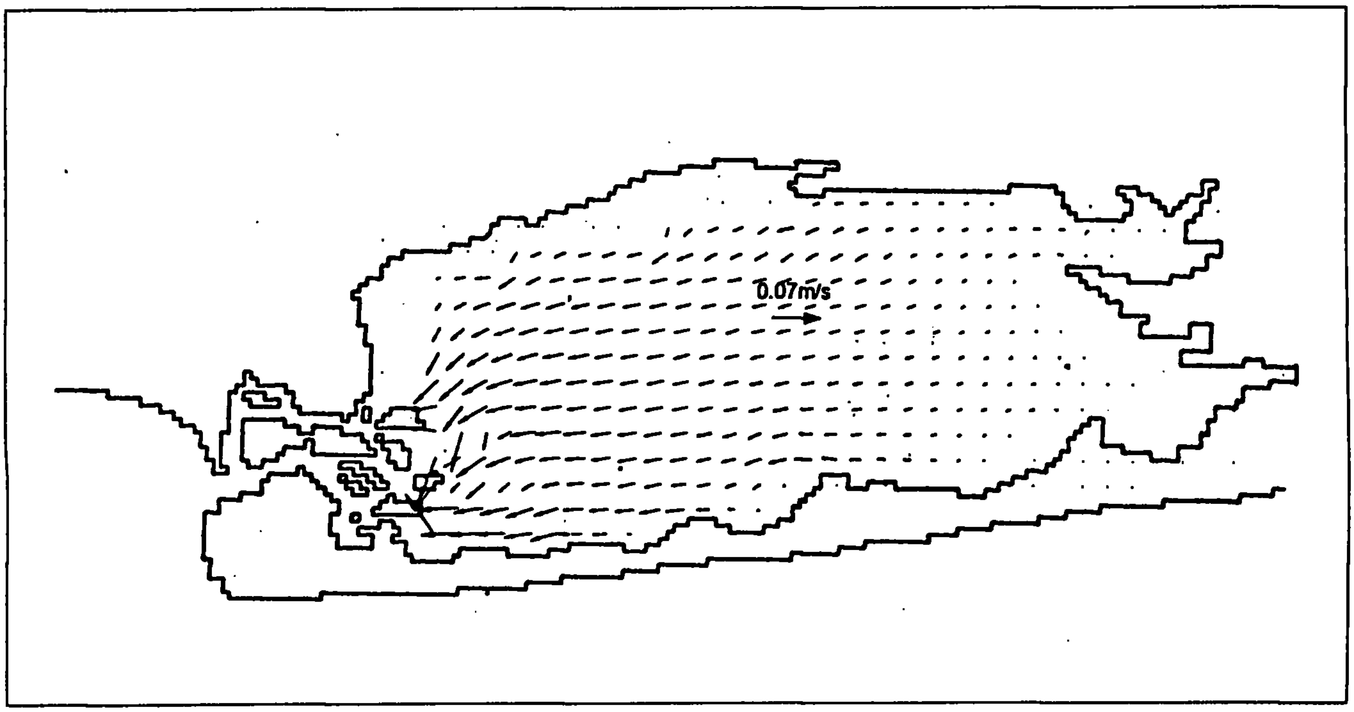

Figure 9. Velocity vector plot in the Lagoon at mean water level on an ebb tide.
(Vector scale given by seperate arrow)

maximum of around 0.6m/s. The Numerically predicted maximum was only around 0.5 m/s, some 16% below the measured velocity. When considering the narrow entrance channels represented by a large grid of 71.43m*71.43m and other approximations in the model this comparison is reasonable.

Even though no measured data were available to validate the predicted residual movements, the results are encouraging. At the far end of the lagoon water movement was relatively small but it should be noted that river discharge was not included for this particular study. It can be seen that the tidally-induced water variations within the lagoon are very small in both absolute and residual motion terms low enough to allow suspended sediments to settle on the bed. Thus it can be expected that the water depths in the lagoon will continue to be reduced which, together with limited circulation of water, may lead to a degradation of water quality with consequent impart on fishing.

5.0 Conclusion

This preliminary study of tidally-induced flows in the Negombo Lagoon gives encouraging results in comparison with the limited amount of calibration data available from the site. The implementation of the results in relation to water quality and sediment deposition may be of significant importance to the future exploitation of the natural resources of the lagoon and therefore need further study, including the collection of water-surface elevation and flow velocity data.

6.0 Acknowledgments

This work constitutes part of the second author's Ph.D. studies at Imperial College with financial support from the Association of Commonwealth Universities which is gratefully acknowledged as in the leave of absence granted to him by the University of Moratuwa. Further work will consider the sediment transport processes due to tide, river and eventually to the lagoon entrance - waves. The core numerical model was developed by Dr. K. Anastasiou, Imperial College and the full scale data was made available by Dr. S.S.L. Hettiarachchi, University of Moratuwa to whom the authors extend their thanks and appreciation.

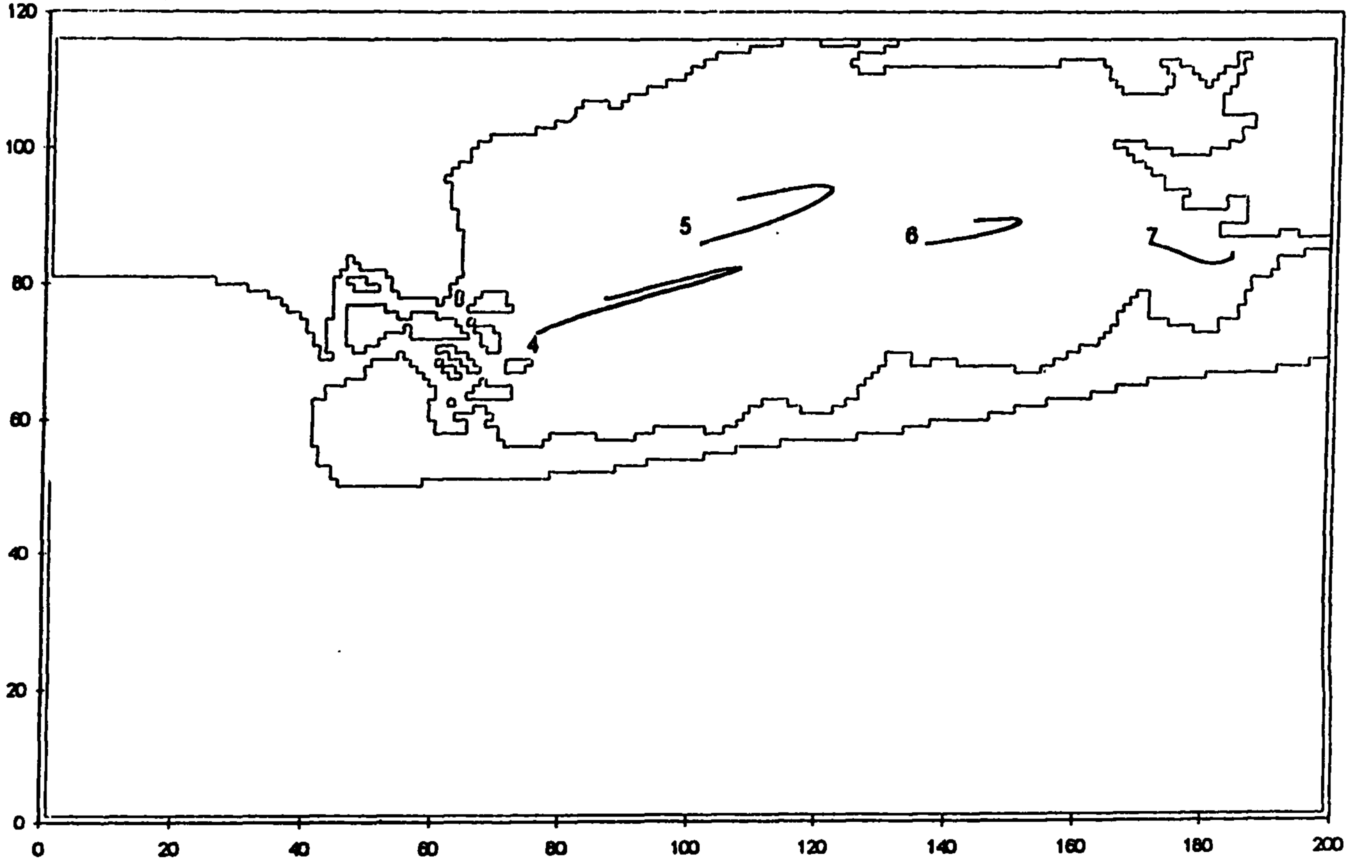

Figure 10. Residual movements over one complete tidal cycle.

7.0 References

Falconer R.A., "Numerical modelling of tidal circulation Harbours", Journal of the Waterway Port and Coastal and Ocean division, ASCE, Vol 106, No. WW1, February 1980.

Falconer R.A. & Owens P.H., "Mathematical modeling of Tidal currents in the Humber estuary", Journal of the Institution of Water Engineers and Scientists, vol. 38, No.6, December 1984, pp. 528-542. Hettiarachchi S.S.L., "Sedimentation in the Negombo Lagoon outlet". Terwel L. & Vermeulen J., "Sedimentation in the Negombo Lagoon". Final year research project report of the Delft University of Technology, conducted in collaboration with the University of Moratuwa, Sept. 1994.