# POWER CURVE REPRESENTATION OF TRANSFORMER MAGNETISATION CHARACTERISTIC

by

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## Introduction

One of the problems encountered in the calculation of transformer transients is the absence of a single mathematical expression to represent the magnetisation characteristic over a wide range of flux. To overcome this problem it is possible to use a point by point representation of the curve with linear or other interpolation between points. However, this would generally lead to a piece wise continuous incremental permeability, or a complex analytical expression. In certain problems, computational instability would also occur. Widger advocates the use of a rational fraction polynomial to obtain a single continuous curve. The method, while giving a fairly good fit for the magnetisation curve, has severe limitations when the incremental permeability is necessary, as, in certain small regions, the incremental permeability could even appear to become negative. The commonly used polynomial function approach too has similar limitations. Macfadyen et al[2] has shown that an exponential series approach does not suffer from the above disadvantage. However, this method may only be used when the flux density B is required, given the magnetic field H, and not vice versa.

The present formulation was necessitated in transient calculations in transformers where the field H had to be repeatedly determined for varying B over almost the complete magnetisation characteristic. The technique developed has been to create a power series of selected indices, such that negative coefficients do not arise, thus avoiding the problems of the derivative. The saturated linear region may finally be represented by a tangential straight line, corresponding to the premeability of free space, meeting the power series curve.

## Form of Equation

Consider the magnetisation curve shown in Figure 1.

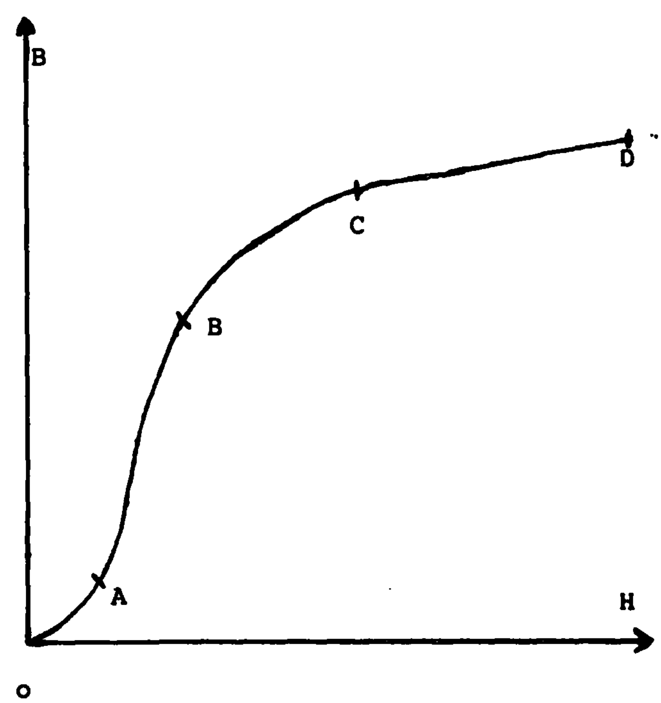


FIGURE 1 - Magnetisation Curve

It is seen that in the initial region OA, the incremental permeability

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$\frac{dB}{dH}$ starts low and increases, while in region AB it is high. In the region BC it starts high and decreases, while in the region CD it is low.

Consider how these regions may be represented.

The initial region of the characteristic may be represented by a positive power of B less than unity.

i.e. $H = k_1 B^{n_1} \qquad n_1 < 1 \qquad (1)$

At high flux densities, the contribution due to this term becomes negligible as H increases rapidly, and the characteristic may be represented by a high positive power of B.

i.e. $H = k_2 B^{n_2} \qquad n_2 \gg 1 \qquad (2)$

The coefficient $k_2$ is small, and thus this term would have negligible effect in the initial region. Thus both regions would be satisfactorily represented by the expression

$$H = k_1 B^{n_1} + k_2 B^{n_2} \qquad (3)$$

An expression to fit the various regions of the curve could thus be obtained by increasing the number of terms successfully chosen. Thus the magnetisation curve may be expressed by a power curve of the form

$$H = \sum k_i B^{n_i}$$

where $k_i$ are positive coefficients and $n_i$ are selected positive indices. The coefficients are deliberately chosen to be positive to ensure that a smooth curve is generated without oscillation.

## Evaluation of Coefficients & Indices

For region OA of Figure 1, we may quite accurately use the representation corresponding to the first term of the series. Thus

$$H = k_1 B^{n_1}$$

The coefficient $k_1$ and the index $n_1$ are determined from the expression

$$\ln H = n_1 \ln B + \ln k_1$$

either by plotting ln H against ln B or by regression analysis. Only the points lying in the linear region of the log-log plot is used in the evaluation, to obtain initial estimates for $k_1$ and $n_1$.

The region AB of Figure 1 may be represented by the first two terms of the series. Thus

$$H = k_1 B^{n_1} + k_2 B^{n_2}$$

or $H - k_1 B^{n_1} = k_2 B^{n_2}$

i.e. $H_2 = k_2 B^{n_2}$

i.e. $\ln H_2 = n_2 \ln B + \ln k_2$

Since $k_1$ and $n_1$ are known, $H_2$ is also known. Thus a log-log plot may again be used to determine $k_2$ and $n_2$, for the linear region of log-log curve. The next region is then used to determine the next pair of coefficients and so on until the required range of B and H values are covered.

Once an initial estimate of the complete curve has been obtained, of the form $H = \sum k_i B^{n_i}$, the parameters are varied using regression analysis to get the best fit. The best fit is obtained by minimizing the sum of the squares of the fractional errors.

## Results

The data for a typical magnetisation curve for Silicon sheet steel was used in the study (vide reference 2). Expressions were obtained to fit the data using (a) the proposed power curve representation (b) a polynomial function and (c) a rational fraction polynomial function, using regression analysis to obtain the best fit for six parameters each.

The best fit, in each case, was defined to correspond to the least square percentage error, rather than the least square error, on account of the extremely large range of H values (from 40 A/m to 80,000 A/m) considered in the study.

These best fits had the respective expressions

$$H = 47.220B^{0.100} + 123.864B^{1.4377} + 7.9063B^{11.372}$$

(for power curve representation)

$$H = 451.37B + 1232.1B^2 - 10590B^3 + 20861.7B^4 - 16210.83B^5 + 4461.1B^6$$

(for polynomial function)

and $$H = \frac{340.02B - 535.69B^2 + 245.01B^3}{1.00 - 0.88698B + 0.1991B^2}$$

(for rational fraction polynomial function)

These are shown in figures 2, 3 and 4 together with the data points on which they are based. (In order to show the extensive range of H considered, in detail, each curve is plotted 4 times on the same graph for decade - extensions of the x - scale).

It is seen that, not only is the fit obtained by the power curve method, suggested in this paper, the best, but also the apparent fit in - between the data points is most regular unlike with the other 2 fits which show irregularities at either the start or at the end. Such irregularities in the BH curve cause, the corresponding gradient, and hence the transformer inductance, to become completely wrong and even negative causing instability in computer calculations of transients in transformer feeders.

Further, in the case of the polynomial and the rational fraction polynomial methods, it is difficult to even obtain a realistic initial estimate of the coefficients, before proceeding onto obtain the best fits. On the other hand, with the proposed method, there lies no such difficulty and the log-log curves described earlier give the initial estimates very easily and accurately. Since this method of selection ensures the use of positive coefficients in the power curve fit, cancellation terms do not occur at any value of B, unlike in the other 2 methods and the curves thus obtained are inherently stable when used in transient calculations.

The method of determining the initial estimates of the parameters for the power curve representation is detailed below. Figure 6 shows a plot of log B vs log H. Since the pair of parameters $k_1$ and $n_1$, are to be determined from the initial region of the B-H characteristic, and further terms are to be added to represent the remaining terms, all data points must lie on the right hand side of the straight line to be chosen on the log-log plot. In this case the line chosen is that corresponding to the first 2 data points only, as shown.

Figure 7 shows a plot of log B vs $\log(H-k_1 B^{n_1})$ without the first few points of the original data. Since the initial region has now been removed, the next term to be added may again be determined by considering the first few terms of the new graph and drawing a straight line on the log-log plot. It is seen that the first 3 points lie on a straight line with the remaining points lying on the right hand side of the line. The parameters $k_2$ and $n_2$ are evaluated from this. Figure 8 shows a plot of log B vs $\log(H-k_1^{n_1} - k_2B^{n_2})$ without the already considered data points. The process is again repeated to determine the next pair of parameters $k_3$ and $n_3$. In this particular example it is seen that the straight line chosen satisfied all the remaining data points quite closely and that no other term is required.

With the 6 parameters thus determined, the initial fit for the B-H curve may be expressed as

$$H = 114.3B^{0.417} + 71.43B^{4.065} + 3.03B^{12.6}$$

which is shown in Figure 5. It is seen that even this, although not necessarily the best fit, is a much better fit than the corresponding best fits from the other 2 methods.

## Conclusion

It is seen from the foregoing analysis, that the technique of power curve fitting with positive coefficients using selected indices, not necessarily integer, for the B-H curve is superior to the commonly used polynomial and rational fraction polynomial functions in accuracy. It is also seen that in this technique, estimation of the parameters for the series may be done in a straight forward manner using log-log plots unlike in the other 2 methods. Also, since the series is formed by the addition of positive terms, the function does not give rise to unstable behaviour when used in the transient analysis of transformer feeders, and in thus recommended for use in such studies when H values have to be determined from known B values.

## References


1. Widgr, C.F.T., "Representation of Magnetisation curves over extensive range by rational fraction approximations", Proc. IEE, Vol. 116, No. 1, Jan. 1969, p 156-160.

2. Macfadyen, W.K. et al, "Representation of Magnetisation curves by exponential series", Proc. IEE, Vol. 120, No. 8, Aug 1973, p 902-904.


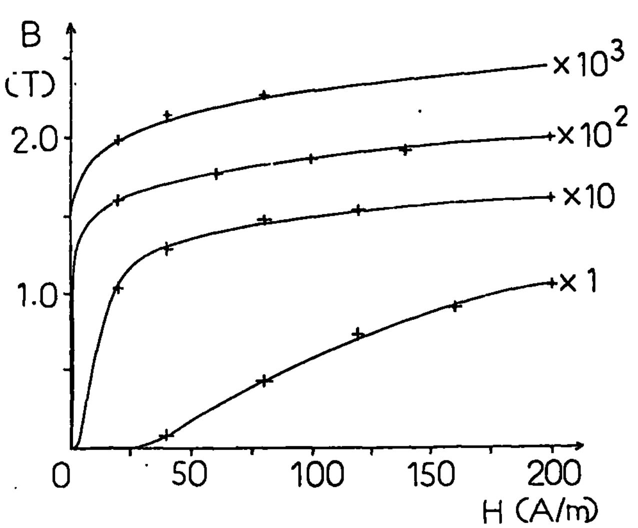


**FIGURE 2 - Using Power Curve Fit (Initial Fit)**

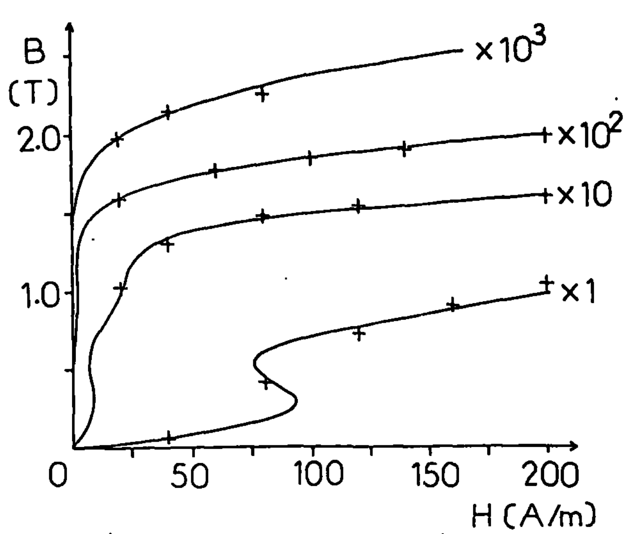


**FIGURE 3 - Using Polynomial Fit (Best Fit)**

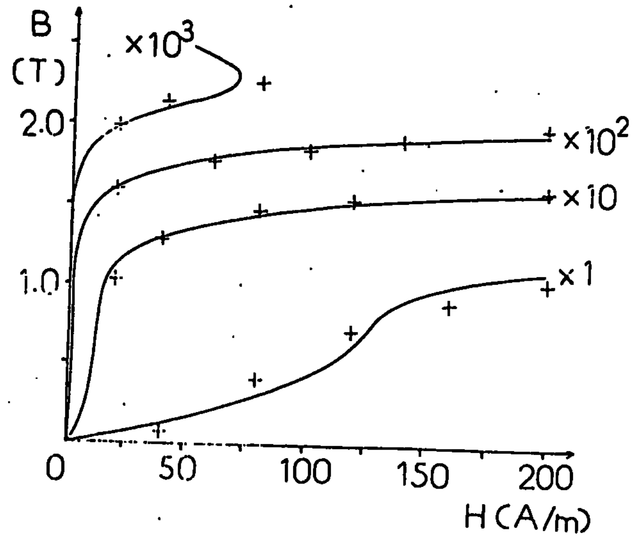


**FIGURE 4 - Using Rational Fraction Fit (Best Fit)**

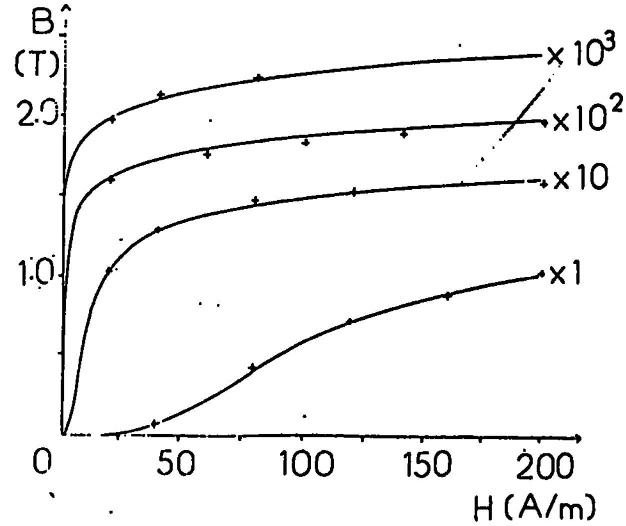


**FIGURE 5 - Using Power Curve Fit (Best Fit)**

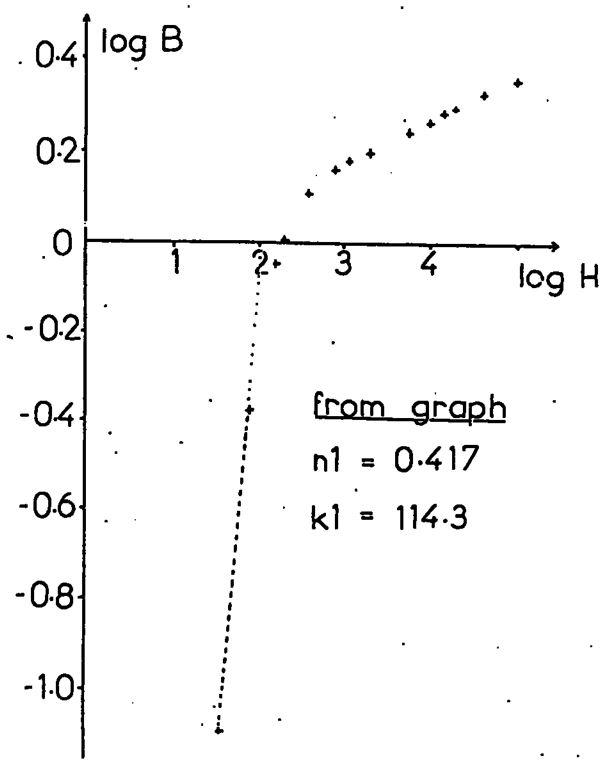


**FIGURE 6 - Graph to Determine $n_1$ & $k_1$**

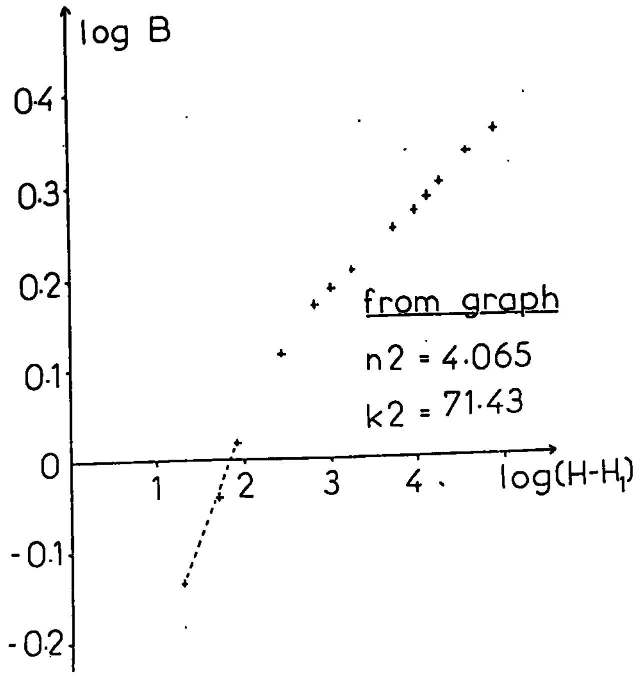


FIGURE 7 - Graph to Determine $n_2$ & $k_2$

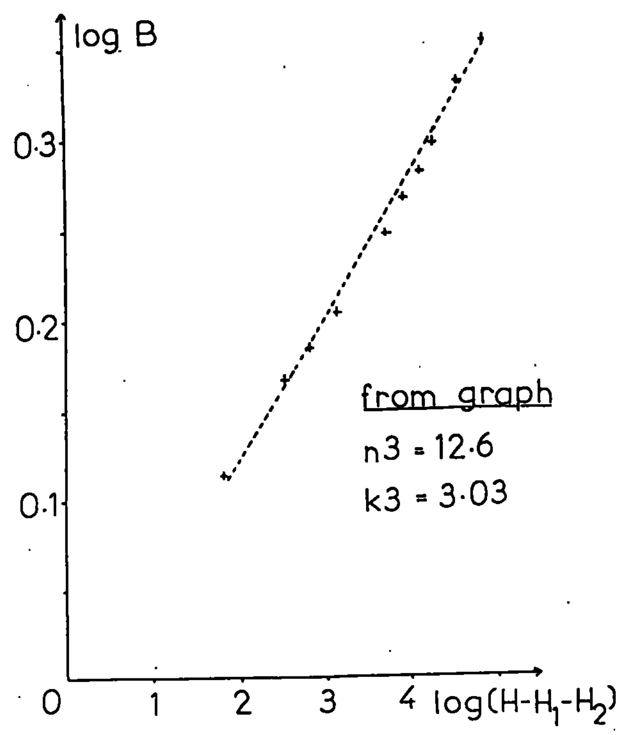


FIGURE 8 - Graph to Determine $n_3$ & $k_3$