

Earthquake Induced Kinematic Forces on Pile Foundations in Layered Medium

H S Thilakasiri, D M Potts, S Kontoe and L Zdravkovic

Abstract: The kinematic forces induced on piles in two-layered soil medium are investigated using finite element method. Typical layered subsurface condition with the soil properties relevant to Sri Lanka are considered in the present study. Hyperbolic nonlinear constitutive model is used for the soil medium and the soil medium is modelled as a fully coupled system. The effect of the earthquake was simulated by lateral ground acceleration applied to the bedrock. The validity of the results of the proposed model was established by comparing with the trends observed in similar studies reported in the literature. The effects of the stiffness of the soil layers, pile diameter, fixity at the pile head, and a defect present at the layer interface on the kinematic forces are investigated in the present study. Finally, the use of the results of the present study in the design and construction of bored and cast in-situ concrete in Sri Lanka are highlighted.

Keywords: Bored piles, Earthquake, bending moment, defective piles

1. Introduction

Failure of pile foundations due to earthquake induced forces are well documented in the literature. Majority of such failures are due to the action of lateral forces applied on the piles due to earthquake induced ground motion. Such failures are always disastrous involving loss of life and property. As a result considerable amount of research work had been carried out at the research institutes throughout the world, especially in the earthquake prone regions of the world, to investigate the development of earthquake induced forces in piles. It is shown, through such research, that the piles are subjected to inertial effects and kinematic effects during an earthquake.

1.1 Inertial Effects of Earthquakes on Pile Foundations

When a structure is supported on a pile, the motion of the pile is transferred to the structure and the structure is forced to oscillate. The oscillatory motion of the mass of the structure generates inertial forces that in turn is applied on the pile and hence on the soil. The deformation of the pile due to the inertial forces may further change the inertial forces in the superstructure. This is referred to as the *inertial response*. Eventhough these two phenomena are occurring simultaneously with a small time gap, they are conveniently considered in two steps. It is proved that the dynamic loading due to the superstructure inertial effects is confined

to a shallow length, termed active length, below the ground surface. The forces applied on the pile foundation due to inertial effects of the structure depends very much on the configuration of the structure, magnitude of the earthquake, distribution of the mass of the structure etc. General design guidelines developed for this purpose may be used to estimate the inertial forces applied on the structure due to an earthquake.

1.2 Kinematic Effects of Earthquakes on Pile Foundations

Eventhough as mentioned previously the inertial effects of the superstructure is concentrated within the active length, generally about ten times the pile diameter, Mizuno (1978) observed failure of piles at depths below the levels attributed to inertial effects. After similar other observations, the research work was focused to find the cause of such failures. Based on the results of the small scale laboratory tests and other numerical and experimental studies, such failures were

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attributed to discontinuities in the subsurface due to sudden variations in the soil stiffness and such effects were termed as kinematic effects of the earthquake on piles.

Due to an earthquake, the soil response at a given site depends on the properties of the soil layers present at the site. The soil response at a given site without any structural element is referred to as the 'free field' motion of the site. If any structural element such as a pile, embedded in the soil medium, is present the ground motion forces it to follow movement of the surrounding soil medium. However due its high rigidity, the piles resist such forced movement and reflect the incident stress waves in the process. This is referred to as the wave scattering and as a results, the ground motion near the pile considerably differs from the free field motion. Due to the forced oscillation, the pile will develop curvature near the layer boundaries due to the difference in the stiffness of the adjacent layers. As a result, the pile is subjected to bending moments and shear forces. This is referred to as the *kinematic response* of the pile. The magnitude of such kinematic forces, developed on piles, mainly depend on the contrast between the layers in the subsurface and have maximum effects near the boundary between the layers in the subsurface (Gazetas and Mylonakis, 1998). The design guidelines available at present take into consideration only the inertial forces applied on the piles due to earthquakes.

Eventhough the development of kinematic forces on pile foundations during earthquakes is by now well accepted among geotechnical earthquake engineering, a rational methodology to estimate such forces on piles is yet to be developed. As the magnitude of the developed kinematic forces depends on the stiffness ratio between the layers present in the subsurface, the development of kinematic forces on piles in highly layered subsurface conditions that exist in Sri Lanka, may be significant. Moreover, there is a high possibility of presence of construction defects, in the vicinity of layer boundaries, in commonly used cast in-situ concrete piles in Sri Lanka. As such, the combined effects of the possible defects and the developed kinematic forces in the vicinity of the layer boundaries may be critical for the stability of the piles in the layered subsurface conditions in Sri Lanka. Therefore, in this research the kinematic forces developed in piles installed in typical layered subsurface

conditions in Sri Lanka are investigated using numerical simulation.

1.3 Literature Review

As mentioned above, various research projects on the kinematic forces developed in piles during earthquakes were conducted. The model tests and numerical simulation have shown that significantly high bending moment is developed within one pile diameter distance from the layer interface. If the pile head is fixed, high bending moment may be developed at the pile head as well. It is observed that the shear forces developed in piles due to kinematic forces are not that significant.

Gazetas and Mylonakis (1998) investigated the behaviour of piles in elastic mediums, subjected to steady state harmonic shear vibration. Gazetas and Mylonakis (1998) used the parameter termed 'active length (L_a)' in the interpretation of their results. They defined the active length in terms of the modulus of the pile material (E_p), modulus of the top soil layer (E_{s1}) and the diameter of the pile (d), as given in Equation [1].

$$L_a = 1.5d \left(\frac{E_p}{E_{s1}} \right)^{0.25}$$

Gazetas and Mylonakis (1998) made the following observations:

- i. The maximum bending moment for free and fix headed piles under steady state harmonic excitation applied at the toe for two different pile diameter are as shown in Figure 1.

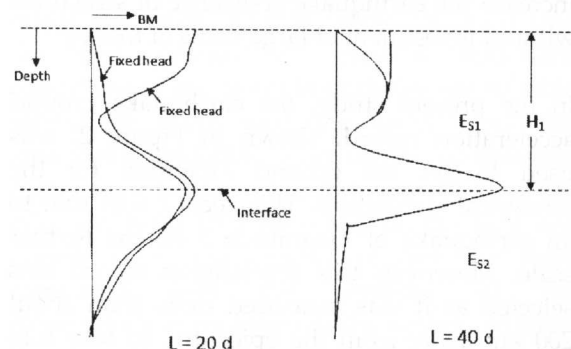


Figure 1 - maximum BM developed in the pile due to steady state harmonic shear vibration (Gazetas and Mylonakis, 1998).

- ii. The maximum moment at the interface for 'free' and 'fixed' head conditions are identical for 'long' piles ($L_a < H_1$);



- iii. The maximum BM at the pile head is more than the interface BM, when ($L_a > H_1$);

Where H_1 is the thickness of the top layer.

2. Realistic Possibility of Earthquake in Sri Lanka

Traditionally it is believed that Sri Lanka is situated well away from boundaries of the tectonic plates and hence safe from earthquakes. Increased tectonic activities associated with the plate boundary near Indonesia and the devastating 2004 Tsunami, created due to one such activity, arose the fear of an earthquake that might hit Sri Lanka. Moreover, numerous earth tremors that were felt across Sri Lanka during the recent past have also created some doubts in the minds of the engineers regarding the traditional thinking of the effects of earthquakes on Sri Lanka. Some geologists argue that there is a new rapture propagating approximately about 500 miles Southeast of Sri Lanka and that has increased the tectonic activities in the region. However, solid proof of such theory is yet to be emerged. As a result of all the hype regarding a possible earthquake hitting Sri Lanka, majority of engineers is of the view that it is better to be prepared for such an event, if it ever happens. As is the case with anywhere in the world, the occurrence of an earthquake is anybody's guess. However, it is better to consider effects of moderate earthquakes during design and construction of structures. Specially, attention should be paid to the additional steps that can be taken during design and construction to increase the earthquake resistance of structures without investment of large sums of money.

In the present study, the earthquake ground acceleration record, shown in Figure 2, was used to get the ground excitation for the numerical simulation. This record was due to an earthquake of magnitude 5 on the Richter scale. Moreover, this acceleration record was selected as it was measured more than about 200 km away from the epicentre, to take into the effects of large epicentral distance to Sri Lanka from an anticipated earthquake. The fundamental period of the earthquake record is 0.85 sec, which is well away from the resonance frequency of the layered system considered in this study. The selection of the earthquake record well away from the resonance frequency of the soil medium was done purposely as the

kinematic effects on the pile is predominant under such conditions.

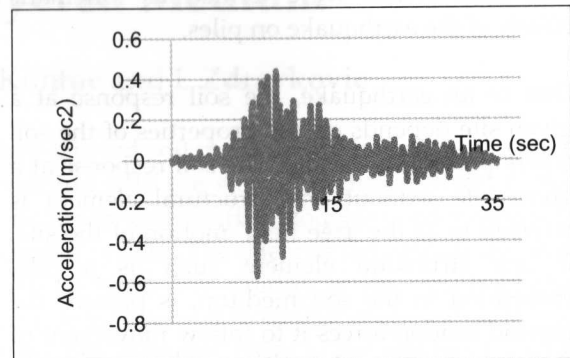


Figure 2 - Earthquake record considered in the present study.

3. Numerical Simulation

3.1 Introduction

Finite Element Computer software, named Imperial College Finite Element Program (ICFEP), developed at the Soil Mechanics section of the Department of Civil and Environmental Engineering at Imperial College, London is used for the present study. ICFEP is specially developed to solve problems related to geotechnical engineering and has been successfully utilized for solving large number of complex geotechnical engineering problems. ICFEP possesses the capability of solving problems related to soil dynamics with the availability of suitable soil constitutive models and the capability of the applying dynamic boundary conditions (Kontoe, 2006).

3.1.1 Soil constitutive Model

The behaviour of soil under dynamic loading is hysteretic, highly nonlinear and plastic. The soil constitutive relationship used in the numerical simulation work should be capable of addressing all these issues, while maintaining a balance between the efficiency and accuracy to handle the problem at hand. Hyperbolic nonlinear constitutive model, shown in Figure 3, is capable of numerically simulating the soil behaviour, when subjected to dynamic forces. In addition, it has the added advantage of well established correlations between simple soil parameters and the damping ratio curves and stiffness decay curves (Guerreiro, 2008). After investigation of the empirical correlations proposed by Vucetic and Dobry (1991), Ishibashi and Zhang (1993) and Darendali

(2001), Guerreiro (2008) recommended the use of the empirical correlation proposed by Darendali (2001) for the damping ratio and stiffness decay curves. Relationship proposed by Darendali (2001) has its applicability limited to cyclic shear strains less than 1.0×10^{-2} and best suited for non-plastic soils and soils with low plasticity index. As most of the soil types present in the Sri Lanka, especially in the built up areas, fall into low plastic category and the low level of the deformation of the ground from an anticipated earthquake, hyperbolic nonlinear constitutive model is used as the soil constitutive model with the empirical correlation proposed by Darendali (2001).

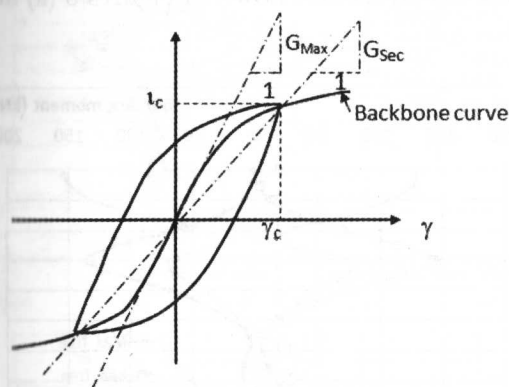


Figure 3 - Hyperbolic nonlinear constitutive model.

3.1.2 Idealization of the pile-soil system

Subsurface consisting of alluvial soft soils overlying strong residual formation commonly encountered in Sri Lanka is used as the subsurface condition. Eventhough the behaviour of a pile is best modelled under three dimensional strain, considering the economy of the solution process plane strain condition was assumed in the model. The subsurface condition with the base soil properties used in the finite element simulation is shown in Figure 4. Linear elastic constitutive relation was used for the pile elements. Joint elements offering limiting shear resistance according to Mohr-Coulomb failure criterion is used to model the interface between the pile and the surrounding soil. The normal and the shear modules of the interface elements were obtained after a sensitivity study.

The thickness of the top loose alluvial deposit was assumed to be 5m and the overall thickness of the subsurface is 20m. The pile at the centre is assumed to penetrate the subsurface and socketed to the bedrock underneath. It is assumed that the water table is present at 1m

below the ground surface level and interface and the soil medium was modelled as a fully coupled system. Drainage was allowed through the lateral boundaries and the bottom boundary was kept undrained. The earthquake excitation was applied to the bedrock as the acceleration in the horizontal direction. The presence of the pile cap or the capping beam on top of the pile was modelled with the extension of the pile 1m above the ground surface and the fixity of the pile at the top, due to the presence of the pile cap or the capping beam, is varied to two extreme cases 'fixed' and 'free'.

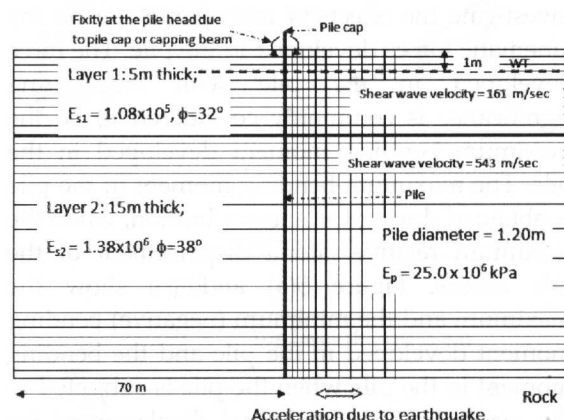


Figure 4 - Two layer system with base soil properties.

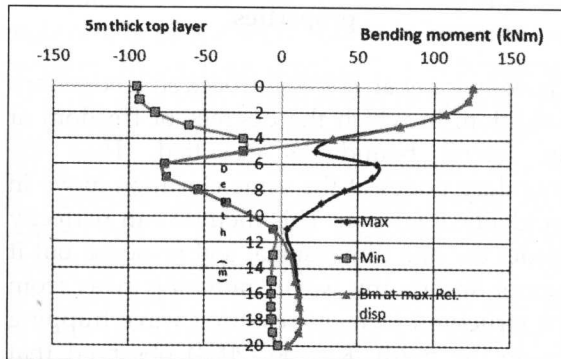
Zienkiewicz et al. (1988) introduced a boundary condition, in which the degrees of freedom at the lateral boundaries are tied. Thus the boundary nodes at the same elevation move in an identical fashion. This method can perfectly model the one dimensional soil response but it cannot absorb any waves radiating away from the structural element resulting wave trapping inside the mesh. Kontoe (2004) observed that the tied degrees of freedom boundary conditions at the mesh boundaries are suitable when the wave radiating away from the structure is negligible. Since there is no measure of the amount of wave radiating away from the pile, the boundary response of the pile soil system with tied degrees of freedom were compared with the response of the corresponding positions of the system without the pile subjected to the same excitation ground motion. The difference between the two systems was brought to acceptable limits by increasing the distance from the pile to the lateral boundary. In this case the material damping of the cyclic soil constitutive behaviour attenuates the wave so that the wave reflected at the boundary becomes insignificant.



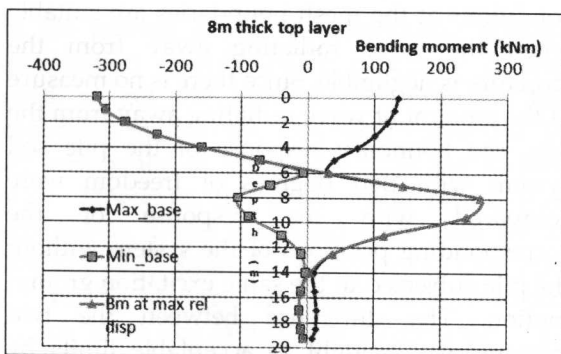
3.2 Implementation of the model

3.2.1 Introduction

A 1.2m diameter pile installed through a two layer soil medium with the top layer thickness 5m and total thickness 20m, shown in Figure 4, is used at the base case. The soil properties shown in Figure 4 are termed in this paper as the base soil properties and the pile-soil system with the soil properties, shown in Figure 4, is termed the base case. Subsequently, thickness of the top layer, stiffness of the layers, fixity at the pile head and the pile diameters were changed to investigate the effects of each parameter on the kinematic forces developed in the pile. The force developed in the pile with engineering significance is commonly considered to be the maximum bending moment developed in the pile. The maximum bending moment in the pile is obtained during the shear vibration, when the maximum relative lateral displacement of the pile occurs. Figure 5(a) and (b) show the maximum and the minimum (negative) bending moment developed in the pile and the bending moment in the pile when the pile is subjected to the maximum relative lateral displacement for top layer thickness 5m and 8m respectively.



(a) 5m thick top layer

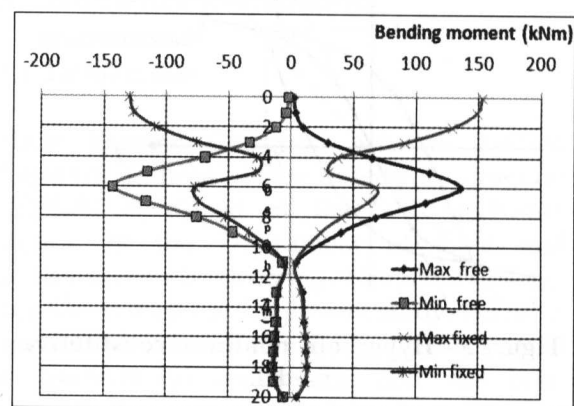


(b) 8m thick top layer

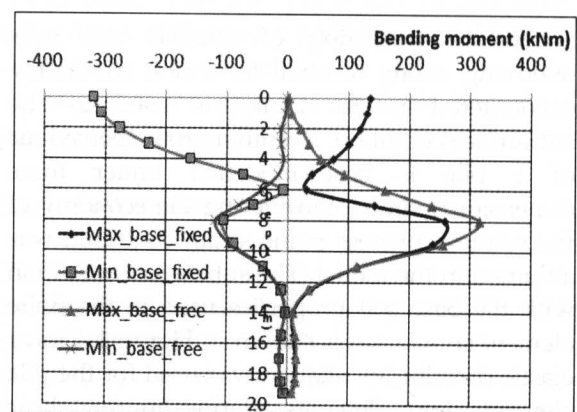
Figure 5 – Maximum and minimum (negative) bending moment in the pile when subjected to the maximum relative lateral displacement: (a) 5m thick top layer; and (b) 8m thick top layer.

3.2.2 Effects of the pile diameter and the fixity at the pile head

The active length, L_a , of 1.2m diameter pile in the subsurface condition shown in Figure 4 (base case) is 7.0m assuming that the initial elastic modulus of the top layer as E_{s1} in Eq[1]. However, it should be noted that the elastic modulus decreases with the shear deformation as shown in Figure 3 and, hence, the active length increases with the increase in the shear deformation of the soil. The maximum and minimum bending moment in the pile for 'fixed' and 'free' head conditions for 5 and 8m thick top layers are shown in Figures 6 (a) and (b).



(a) 5m thick top layer



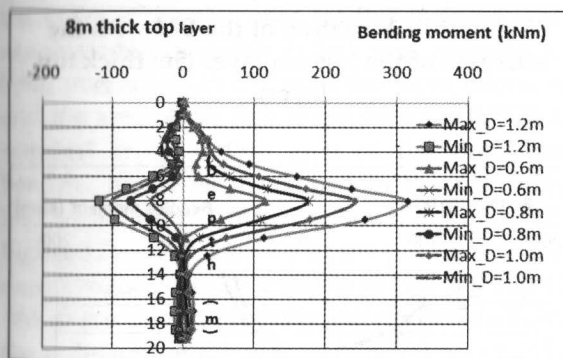
(b) 8m thick top layer

Figure 6 - The maximum and minimum bending moment in the pile for 'fixed' and 'free' head condition.

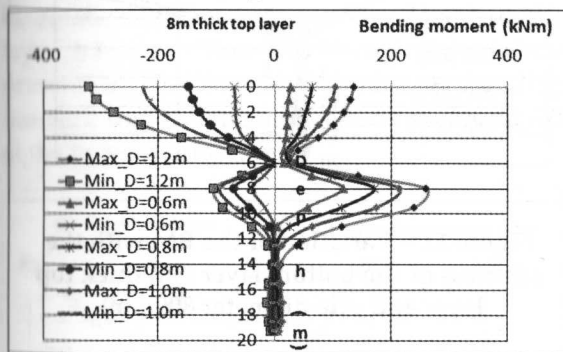
For Figure 6(b) top layer thickness is 8m ($> L_a = 7m$), hence the interface bending moment for the free headed pile and that for the fixed headed pile are very close as concluded by Gazetas and Mylonakis (1998). However, when the top layer thickness is 5m ($< L_a$), the interface bending moment is less than the pile head

bending moment. The pile head bending moment is more than the interface bending moment for top layer thickness 5m ($< L_a$) but not for top layer thickness 8m ($> L_a$) as observed by Gazetas and Mylonakis (1998).

The diameter of the pile is varied (1.2m, 1.0m, 0.8m and 0.6m) for 8m thick top layer for 'free' and 'fixed' head conditions are shown in Figures 7 (a) and (b) respectively.



(a) Free headed pile

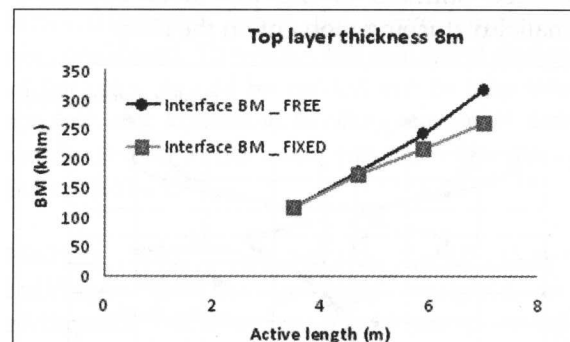


(b) fixed head pile.

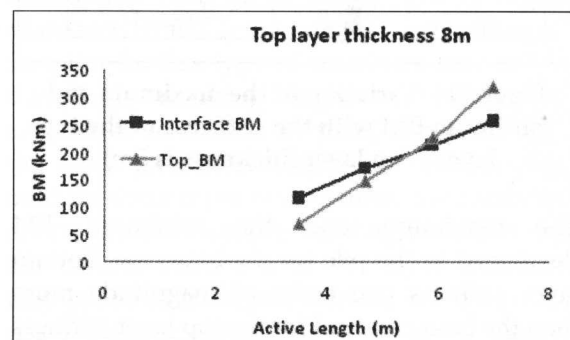
Figure 7 - Maximum and minimum bending moment for different pile diameters.

It is observed that both the pile head BM and the interface BM increases with the increase in the pile diameter. Interface BM for 'free' and 'fixed' headed piles against the corresponding active lengths were plotted in Figure 8(a), and the top BM and the interface BM against the active lengths are plotted in Figure 8(b). Interface BM for 'free' head condition is more than the BM for the 'fixed' head condition for the active length higher than about 5.5m. Similarly, the top BM is more than the interface BM for 'fixed' head condition when the active length is more than about 5.5. However, since the top layer thickness is 8m, the above change should take place at the active length 8m. The active lengths were calculated using the initial elastic modulus of the soil and the reduction in the elastic modulus with the shear strain was

not considered. The reduction in the soil modulus may have resulted in the increase in the actual active length of the piles than the ones used in plotting Figure 8. Therefore, the observations of Gazetas and Mylonakis (1998) may be still applicable to transient dynamic vibrations due to earthquakes but the modulus reduction with the shear strain should be considered in the estimation of the active length.



(a)



(b)

Figure 8 - (a) BM at the interface for 'free' and 'fixed' head condition vs. Active length; and (b) Top BM and interface BM vs. Active length.

The kinematic bending moments obtained from the proposed model show the trends reported in the literature for similar numerical and experimental simulations. Therefore, one could argue that the proposed model may accurately predict the kinematic behaviour of piles subjected to earthquakes.

3.2.3 Effects of the stiffness of the soil medium

To investigate the effects of the stiffness of the layers on the kinematic bending moment developed in the pile, the elastic modulus of the top and the bottom layers were varied. The maximum and the minimum BM developed in the pile for the base case, top layer stiffness one order of magnitude less than the base case and

for the top layer stiffness equals to that of the bottom layer are shown in Figure 9. Figure 9 clearly shows the increase in the BM near the layer interface, when there exists a stiffness difference between the layers. Further, the reduction in the stiffness of the top layer has not resulted in a change in the BM at the interface. However, the BM at the pile head was reduced due to the reduction of the top layer stiffness. Such a reduction is expected, as the reduced stiffness of the top layer applies a smaller disturbing moment on the fixity.

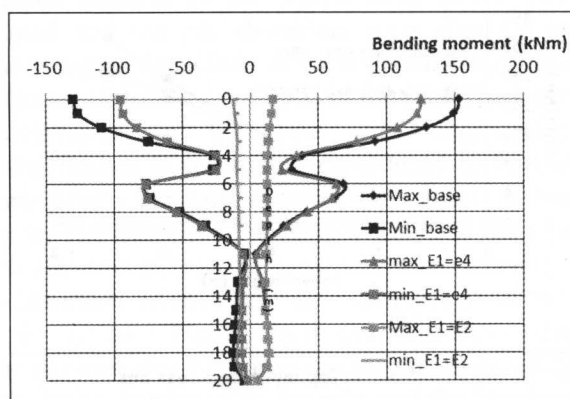


Figure 9 - Variation of the maximum and minimum BM with the stiffness of the top layer (Top layer thickness is 5m).

The maximum and the minimum BM developed in the pile for the base case, bottom layer stiffness one order of magnitude more than the base case and for the top layer stiffness equals to that of the bottom layer are shown in Figure 10. The thickness of the top layer is 5.0m ($H_1 < L_a = 7.0m$). Therefore, increase in the stiffness of the bottom layer, reduces the BM in the pile. The maximum and the minimum BM developed in a 800mm diameter pile for the base soil properties and bottom layer stiffness one order of magnitude more than the base properties are shown in Figure 11. The thickness of the top layer is 8m ($H_1 > L_a = 4.7m$) and hence the entire active length of the pile is in the top layer. The one order of magnitude increase in the soil modulus of the bottom layer does not show a significant reduction in the bending moment developed at the layer interface.

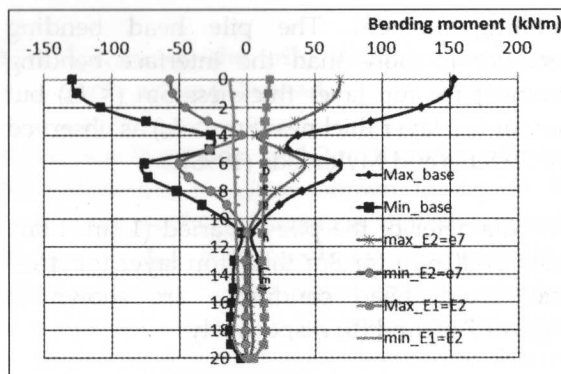


Figure 10 - Variation of the BM with the stiffness of the bottom layer (5m thick top layer).

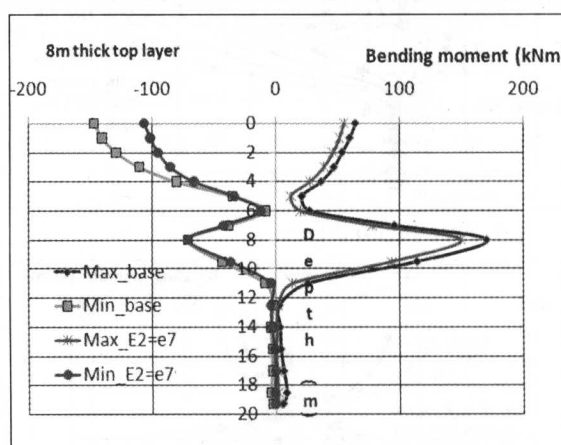


Figure 11 - Variation of the BM with the stiffness of the bottom layer (5m thick top layer and pile diameter 800mm).

In general, comparison of the reduction in the bending moment due to same increase in the stiffness of the bottom layer, shown in Figures 10 and 11, clearly show that the effect of the stiffness increase of the bottom layer has a significant reduction in the bending moment at the interface, when the top layer thickness is less than the active length ($H_1 < L_a$).

3.2.4 Effects of a defect present near the interface

Construction of bored and cast in-situ bored piles are done by drilling a hole in the ground and filling it with concrete after inserting a reinforcement cage. Usually, only the top few meters of the pile bore is supported with a casing and bentonite slurry is used to stabilize the borehole sides. During concreting the bentonite in the pile bore is systematically removed with fresh concrete. However, if proper quality control measures are not used, the pile shaft may be defective and very often

these defects are present near the layer interfaces creating a weak section of the pile near the layer interface.

To investigate the effect of a defective pile section at the layer interface on the kinematic bending moment developed in the pile, a pile with reduced cross sectional area at the layer interface was simulated with the finite element model. The two layer system with the base soil properties, given in Figure 4, and a top layer thickness of 8m was used for this purpose. The nominal pile diameter (1.2m) was reduced to 0.9m over a length of 3m at the layer interface and the area of the pile at the defect is 0.5625 x nominal cross section area of the pile. The bending moments developed in the pile with and without the defect are shown in Figure 12. Figure 12 clearly shows that the presence of the defect reduces the magnitude of the developed BM in the pile at the layer interface. However, the reduction of the pile stiffness by about 44% has resulted in only about 25% reduction of the kinematic bending moment developed at the layer interface. Therefore, even with defects present at the interface significant bending moment due to kinematic effects is developed at the layer interface.

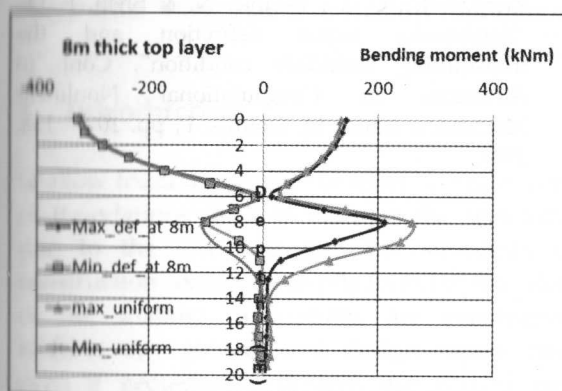


Figure 12 - BM with and without defect at the layer interface.

4. Practical applications in Sri Lanka

Rock socketed end bearing bored piles are commonly used in Sri Lanka. The subsurface condition encountered in Sri Lanka is generally layered with contrasting stiffness and very often, can be idealized into a two layer system with a relatively soft layer overlying a strong residual formation. The subsurface generally consists of silty clayey sand except in places, where very soft compressible organic deposits are present near the ground surface. The results presented here were obtained generally for the

base soil properties given in Figure 4. The pile diameter was varied in the presented study from 0.6m to 1.2m covering the general sizes of the bored piles used in Sri Lanka.

High-rise buildings are usually analysed for earthquake induced resonance conditions and the foundations are designed for inertial effects higher than the kinetic effects. However, these inertial forces are assumed to concentrated only within the active pile length and any kinematic forces developed below the active length are not considered. Therefore, the structural design of the piles should be carried out to take into account any kinematic bending moments that are developed at the layer interfaces present in the subsurface.

Medium size structures (5 to 10 storey buildings) are not generally designed for earthquake induced forces and the piles are designed only to carry axial forces without considering earthquake induce lateral forces. Moreover, construction quality controlling of the piles for this type of structures may not be that high resulting higher probability of occurrence of defects in piles, especially near the layer interfaces. Therefore, for the piles, used for these types of structures, the combined effects of the kinematic bending moments and the defects present near the layer interfaces may prove to be highly critical. To reduce the risk of failure of such piles, enhanced quality control programs should be implemented during construction of the piles for these types of structures. Furthermore, the piles should be designed to carry a nominal bending moment to take into account probable kinematic effects due to earthquakes. Design guidelines should be changed to include such nominal bending moment, after further study on this matter.

The present study again emphasises the importance of having a defect free pile shaft. This could be achieved by putting in place a comprehensive quality control program without significant additional investment.

5. Conclusions

Effects of the kinematic forces in a pile, installed through a layered medium, due to earthquakes are investigated considering typical two layer subsurface condition encountered in Sri Lanka. The results of the finite element simulation presented in this study clearly show the presence of kinematic forces in the pile near the

layer interface. The kinematic bending moment developed near the layer interface increases with the diameter of the pile. The fixity at the pile head increases the bending moment at the pile head and slightly reduces the bending moment in the pile near the layer interface. The results of the finite element simulation show similar trends as the results of the other simplified studies reported in the literature.

It is shown that the location of the layer interface in relation to the active length of the pile is an important parameter in the development of the kinematic forces in the pile. However, it is shown that the stiffness decay of the soil layers increases the actual active length more than the active length estimated assuming initial elastic modulus of the top layer. The BM at the pile top and the interface are significantly different when the top layer thickness is less than the active length. The reduction in the stiffness of the top layer reduces the bending moment at the pile top while the bending moment at the interface is almost unaltered due to such reduction in the stiffness of the top layer. If the bottom layer is below the active length, its stiffness doesn't have a significant effect on the kinematic force in the pile.

A defect present near the layer interface reduces the kinematic bending moment developed. However, even with a defect near the layer interface, a kinematic bending moment of significant bending moment is developed. This emphasises the need for having a uniform pile shaft to increase the earthquake resistance of piles. The results of the present study demonstrate the need for structurally designing the entire pile shaft to carry a nominal bending moment considering kinematic bending moment possible due to an earthquake.

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