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AN INVESTIGATION OF A PROBLEM RELATING TO THE CALIBRATION OF BALANCES

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Introduction

Man first judged the weight of goods by the load he could carry, just as he assessed length by the breadth of his finger or his palm or by the length of arm. As he developed, made things and traded in them, he had to find more accurate means of testing the weight of goods, commodities and persons.

The first instrument used to compare the weights of two bodies was the beam scale. This consists of a rigid beam, balanced at the central point (the fulcrum), with the loads suspended from each end of the beam. The urge to know and measure weight more and more precisely, has led to the whole series of developments from wooden beam scale to mechanical counter scales and steelyards, (compound-lever machines), crane machines, load cells and finally to sophisticated electronic weighing scales.

It is a well known fact that no nation can be self-sufficient to meet all its requirements. All the nations have to import certain commodities and also export materials and products. For international trade the weighing instrument plays an important role, which ultimately helps in the development of the economy of the country. The industrial products of a country may cover areas such as agriculture, textiles, building and construction, heavy engineering, transport and communication, electrical and electronic equipment, consumer goods etc. For standardisation, quality control, interchangeability of parts, safety etc., it is very essential that industrial products, apart from being tested for their performance and durability by the manufacturer during the production process, are evaluated by an independent testing agency which gives a quality mark or certificate. These testing operations are carried out using weighing and measuring instruments of certified accuracies. Apart from uniformity in test procedures, weighing and measuring instruments should have accuracies traceable to the national standards of measurements of that country.

Procedure adopted for calibration balances by H. E. Almer, a metrologist is based on an assumption that the accuracy of a weighing instrument is determined by the precision with which readings were obtained. It is true that the precision, which depends on temperature, pressure, humidity, fluctuation of the temperature difference between and knife edges, rate of change of bending of the beam at a given load, finite size of knife edges, frictional forces acting at knife edges and bearings; wear and tear of the moving parts, vibrational and mechanical shocks (distrosion), thermal and heat waves, arrestment system etc., is a good quality to be appreciated. But when testing a balanee errors due to inequality of the arms, finite size of knife edges, assymmetry of the balance, (DCO should be perpendicular to AB figure is on Appendix I) and lack of sensitivity may create equally or more serious defects in the balance. Error due to the finite size of the knife edge is minimized by designing the knife edge perfectly sharp, horizontal and parallel to each other. Assymmetry of the balance is justified because the balance is highly symmetrical and CD and OD are extremely small compared with AB.

Weighing instruments are classified into four groups according to the accuracy of the balance. Sensitivity of a balance is it's ability to detect the difference between two weights. Thus sensitivity,(1)

$$\mu = \frac{l}{GS + 2a\,(P + Q)}$$

Where G - mass of the beam
S - distance between D and O
a - distance between D and C
P - mass of pan
Q - mass on pan
l - arm length (AC)

2. ALMER'S Procedure for Testing Balances

The balance is tested by placing two weights according to a schedule[5] and observations were obtained at O-load, (no load on pans) at load M_1 and at load M_2 respectively. This series of measurements gives 12 indications at zero load, 10 indications at load M_1 and 10 indications at load M_2. Standard deviations of measurements at O-load, load M_1 and M_2 are determined and each set of observations were analysed by F-test. If the ratios are insignificant at 95% confidence limits the pooled estimator of standard deviations is calculated. This procedure is known as starting sigma. See references 5 or 6 for complete text.

2.1 Comments on Almer's Procedure
Part I—Single Pan Balances

As the name suggests the single pan balance has only one pan. To weigh an object, the load is placed on the pan, and by removing appropriate weights from an array supported by its stirrup equilibrium is restored. Special provision has been made with the help of an optical projection system and a graduated graticule to assess residual difference between the object and the removed weights.

When an observer weighs an object several times using the same balance he may not always record exactly the same measurement. This may result from some lack of precision or uniformity of the weighing instrument used, or from the variability of the observer, or from some small changes in other physical factors which control the measurements. Errors of this nature are known as systematic errors.

Sensitivity of a balance varies with the load on pans. The variability depends on the phase (a-distance between C and D) of knives. Considerable advantages that a single pan balance has over a two pan balanceis its special design which provides a fixed sensitivity throughout the weighing range. This enables the observer to assess the residual difference between loads with the help of the graticule irrespective of load on the pans.

Standard deviation measures the precision or the repeatability of a balance. This should be a fixed quantity in the weighing range as the balance has designed to assess the weight irrespective of the load on the pan

$$\text{ie} \quad S_1 \backsimeq S_2 \backsimeq S_3$$

Where S_1 S_2 and S_3 are standard deviations of the balance at zero load, load M_1 and load M_2 respectively.

The above fact can be established by the following experiment.

2.1.1 Experiment

The balance was released 20 times consecutively and the corresponding rest points were obtained at a given load. This process was repeated at different load levels. The values obtained are said to have come from a normal population.(1)

Data obtained from three single pan balances are given below. Results pertaining to balance number 3 was obtained from Almer's experiment.(1)

Balance Number	S_1 mg	S_2 mg	S_3 mg
1	0	0	0
2	0·399	0.426	0·441
3	0.102	0·083	0.095

Number 2 balance was a rejected one as it was not capable of repeating even at 1 mg while it should have repeated at 0.1 mg according to the manufacturers specifications. But experimental results show that the standard deviations calculated at different load levels are equal to each other

$$\text{ie} \quad S_1 \backsimeq S_2 \backsimeq S_3$$

F-test is used to determine whether the variability of one set of data is significantly different from the variability of another set. For example F-test can be used to test whether the balance number (1) gives more consistent results than that of balance number (2).

Almer's calibration procedure suggests a method which use the F-test to determine whether S_1, S_2 and S_3 has obtained from one set of data. But S_1, S_2 and S_3 are standard deviations of a balance at different load levels. They are equal to each other and represent the same population.

According to Almer's method if the following conditions are satisfied then the balance is accepted as a precise instrument,

$$\frac{S_1^2}{S_2^2} < 3.10, \quad \frac{S_1^2}{S_3^2} < 3.10 \text{ and } \frac{S_3^2}{S_2^2} < 3.18$$

But since

$$S_1 \backsimeq S_2 \backsimeq S_3$$

$$\text{always} \quad \frac{S_1^2}{S_2^2} \backsimeq \frac{S_2^2}{S_3^2} \backsimeq \frac{S_3^2}{S_1^2} \backsimeq 1 < 3.10$$

According to the above test the rejected balance (balance number 2) is a precise instrument which is not correct.

ie. The Almer's calibration procedure suggests that the balance is precise whether the performance of the instrument is accurate or not.

2.2 Part II – Double Pan Balance

Calibration procedure adopted for double pan balance. The testing procedure is similar to the single pan balance but two loads M_1 and M_2 are interchanged according to the schedule. M_1 and M_2 are two masses.

Let us examine the effects of the interchangeability test to precision of the balance.

Suppose X_{0i} be the rest point of the balance when there is no load on pans, X_{1i} be the rest point of the balance when loads M_1 and M2 are placed on pans and X_{2i} be the rest point of the balance when M_1 and M_2 are interchanged. Then the position of the rest point at different load levels will be as follows.

O-Load	Load M_1 & M_2	Load M_2 & M_1	F_1^*	G_1^* (page 9)
X_{01}	X_{11}	X_{21}	—	—
X_{02}	X_{12}	X_{22}	—	—
X_{03}	X_{13}	X_{23}	—	—
—	—	—	—	—
—	—	—	—	—
X_{0i}	X_{1i}	X_{2i}	$\frac{X_{1i}-X_{2i}}{2}$	—
—	—	—	—	—
—	—	—	—	—
X_{0n}	X_{in}	X_{2n}	—	—

where $F_i = \dfrac{X_{1i} - X_{2i}}{2}$ and

$$G_i = \frac{(X_{1,2i-1} + X_{1,2i}) - (X_{2,2i-1} + X_{2,2i})}{4}$$

F_i and G_i are given in mass units.

If the two arms of the balance are not equal to each other and as a result of which the pointer (X_{2i}) goes out of the scale, then massm_0 is added to bring the pointer back to the scale. This process is repeated throughout the experiment. Then the readings will be as follows.

Load$_{M_1}$ & M_2	Load$_{M_2}$ & M_1	$2F_i$	$2F$
X_{11}	$m_0 + X_{21}$	$m_0 + X_{21} - X_{11}$	$m_0 + X_{31}$
X_{12}	$m_0 + X_{22}$	$m_0 + X_{22} - X_{12}$	$m_0 + X_{32}$
X_{13}	$m_0 + X_{23}$	$m_0 + X_{23} - X_{13}$	$m_0 + X_{33}$
-	-	-	-
-	-	-	-
-	-	-	-
X_{1i}	$m_0 + X_{2i}$	$m_0 + X_{2i} - X_{1i}$	$m_0 + X_{3i}$
-	-	-	-
-	-	-	-
X_{1n}	$m_0 + X_{2n}$	$m_0 + X_{2n} - X_{1n}$	$m_0 + X_{3n}$

$$2\sum_{i=1}^{n} F = nm_0 + \sum_{i=1}^{n} X_{3i}$$

$$2F \text{ mean} = mo + \bar{X}_3$$

$$\text{where } X_{3i} = X_{2i} - X_{1i} \text{ and } X_3 = \sum_{i=1}^{n} \frac{X_{3i}}{n}$$

$$\text{standard deviation} \quad S = \sqrt{\frac{\sum_{i-1}^{n}\left(m_0 + ete - \bar{X}_3\right)^2}{n-1}}$$

$$S = \sqrt{\frac{\sum_{ai}^{n} (X_{3i} - \bar{X}_3)^2}{n-1}}$$

Since the expression for s is independent of m_0, standard deviation of the balance is independent of the inequality of the arms. Therefore interchangeability has no effect on the precision of the instrument.

2.3 Conclusion

According to the Almer's analysis the balance is precise if the ratio of variance is less than 3.10. But mathematical analysis shows that the ratio of variance always tends towards unity irrespective of whether the balance is precise or not.

Similarly interchangeability test does not measure the precision of the double pan balance. Therefore F-test is not suitable to analyse the experimental results.

For double pan balances it is mentioned that "the measurement outlined above will give a fair indication of a balance ability to repeat its indication" at different load levels. But a method of obtaining a fair indication was not specified. Thus the Almer's procedure has led to a subjective devision.

Suggestion : F-test does not give correct indication of the precision of the instrument. Therefore one has got to look for an alternative method to get at the correct precision of the instrument. Correct method to analyse the experimental results is t-test which is carried out as follows. Even though this method has designed to test two pan balances it can be adopted to test the performance of single pan balances.

3. Double Pan Balance

The basis of mass calibration is that a weight of known mass is compared with a weight whose mass has to be determined. The process implies a comparator, and the comparator in this instance is the balance. As the accuracy of the balance governs the accuracy of the measurement it is necessary to know the performance of the balance, ie. its sensitivity, precision, stability of the rest point, etc.

Quality and the precision of a balance can be judged by releasing it a number of times. This is done first at the mark capacity and then it is repeated at different capacities. The balance is loaded with two nominally equal weights and the equilibrium of the balance is restored by adding fractional weights as required. Then the rest point is obtained. The sensitivity of the balance is evaluated by adding a small known weight. The masses are then transposed and the rest point is obtained. The above process is repeated according to the following table. In the table M_1 and N_2 are load on pans when the instrument is balanced. $8\Delta m$ represents a small weight whose mass is known.

TABLE 3

TRANSPOSITION WEIGHING

Time Observation No.	Load of pans left	right	rest point (in scale divisions)
1	M_1	M_2	a_1
2	M_1	M_2+sm	a_2
3	M_1	M_2	a_3
4	M_1	M_2+sm	a_4
5	M_1	M_2	a_5
6	M_2	M_1	a_6
7	M_2	M_1+sm	a_7
8	M_2	M_1	a_8
9	M_2	M_1+sm	a_9
10	M_2	M_1	a_{10}
11	M_1	M_2	a_{11}
12	M_2	M_1	a_{12}
13	M_1	M_2	a_{13} e
14	M_2	M_1	a_{14}
15	M_1	M_2	a_{15}
16	M_2	M_1	a_{16}
17	M_1	M_2	a_{17}
18	M_2	M_1	a_{18}
19	M_1	M_2	a_{19}
20	M_2	M_1	a_{20}
21	M_1	M_2	a_{21}
22	M_2	M_1	a_{22}
23	M_1	M_2	a_{23}
24	M_2	M_1	a_{24}

The above series of weighing gives,

10 — observations at load M_1 and M_2
10 — observations when load M_1 and M_2 are transposed.

The readings observed with the addition of the sensitivity weight enable the conversion of the scale divisions into mass units.

The value of a scale division is determined using the relation,

$$1 \text{ scale division} = \frac{\Delta m}{a}$$

$$\text{where } a_0 = a_1 - a_2 + |a_2 - a_3| + |a_3 - a_4| + |a_4 - a_5| + | \; |a_6 - a_7| + |a_7 - a_8| + |a_8 - a_9| + |a_9 - a_{10}$$

Balances are classified into different classes according to the sensitivity of the balance. The sensitivity of a balance at a given load and the class of accuracy which it belongs can be obtained from tables. (Eg. OIML recommendation No. 3). If the experimental value is less than or equal to the table value then it is accepted that the sensitivity of the balance has reached it's standard. Otherwise the balance is rejected at this stage.

Let $\bar{X}_1$ and S_1 be the mean and the standard deviation of the series of observations when two weights are placed on pans. $\bar{X}_2$ and S_2 be the mean and the standard deviation when the weights are interchanged, n_1 and n_2 be number of observations obtained respectively.

If the two arms of the balance are of equal length then $\bar{X}_1 = \bar{X}_2$. This is an essential condition for a high quality balance. But it is not easy to achieve this condition because of the difficulties inherent in the construction process. As such general practice is to define a limit of tolerance for the difference $(\bar{X}_1 = \bar{X}_2)$. Tolerances which depend on the class of accuracy of the balance are given on tables (eg. OIML No. 3).

T-test can be used to compare the difference between the specified tolerance and the difference between lengths of the arms. If λ be the tolerance specified for a particular class of accuracy,

$$\text{Then } t_{n_1 + n_2 - 2} \quad \frac{\lambda - \bar{X}}{2\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\text{where } \bar{X} = |\bar{X}_1 - \bar{X}_2|$$

$$\text{and } \quad S = \sqrt{\frac{(n_1 - 1)\, S_1^2 + (n_2 - 1)\, S_2^2}{n_1 - n_2 - 2}}$$

If $\lambda > \bar{X} + t_{n_1 + n_2 - 2} \; S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ then it is accepted that the inequality and the stability of the rest point is within the limit of tolerance. Otherwise it is rejected.

An experiment was carried out using two 2 kg weights and a 20 mg weight as M_1, M_2 and ΔM and the following corresponding values are respectively obtained for a_i ($i = 1$ to 24).

2.00	1.28	2.02	1.30	1.98	1.92
1.21	1.97	1.24	1.93	1.97	1.96
1.98	1.95	1.96	1.96	2.02	1.97
2.00	1.97	2.02	2.01	2.00	2·03

Then the following computations were done from the above set of observations.

One scale division of the balance = 27.83 mg

The rest point X_1 of the balance and the standard deviations S_1 are

$X_1 = 1.995 \qquad S_1 = 0.0217$

Similarly X_2 and S_2 of the balance when the load is transposed are

$X_2 = 1.967,\ S_2 = 0.033$

Number of observations obtained $n_1 = n_2 = 10$.

The pooled standard deviation S of S_1 and S_2 in mass units is given by

$$\bar{S} = \sqrt{\frac{10 \times 0.0217^2 + 0 \times 0.033^2 \times 27.83}{10 + 10 - 2}}$$

$= 0.8193$ mg

Then $\overline{X} = (1.995 - 1.967)\ 27.83$

$\overline{X} = 0.779$ mg

Condition for acceptance

$$\text{If } \lambda > \bar{X} + t_{n_1 + n_2 - 2}\, S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

then it is accepted that the errors associated with the balance are within the limits of tolerance.

The value of $t_{n_1 + n_2 - 2}$ at 99% confidence limit = 2.552

The value of λ for class II balance of 5 kg. capacity = 30 mg

$$\therefore \lambda = 0.779 + 2.552 \times 0.8193 \sqrt{\frac{1}{10} + \frac{1}{10}}$$

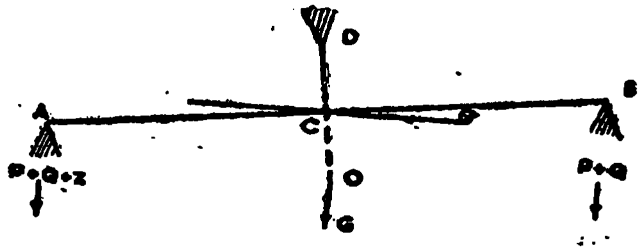
$= 1.71$ mg

To test whether the errors associated with the balance are within the limit of tolerance; computed value—of 1.7 mg is compared with the table value 30 mg. Since the computed value is much less than the table value performance of the balance can be accepted.

If K is highly significant which is due to the inequality of the arms then the experiment should be repeated afer making the proper adjustments of the arms. If the standard deviation is significant then it may be due to the following defects: Viz. chipped or wornout knives, scored bearings, non parallel knife edges, improper arrestment mechanism, random errors, or personal error of the operator. Then each of the defects should be rectified independently. The experiment analyses all the errors associated with a balance to a high degree of accuracy and is applicable to all classes of balances.

APPENDIX I

Theory of the Equal Arm Balance



Where D = fulcrum knife edge
A& B = terminal knife edges
C = mid point of AB (AB – 2l)
O = Centre of gravity of the beam along

P = mass of pan—suspension
Q = mass on right pan
Q + Z = mass on left pan
θ = angle of inclination of the beam to the horizontal

By taking moments of the vertical forces about the fulcrum D when the balance is in the position of static equilibrium it can be shown that the sensitivity

$$\mu = \frac{1}{GS + 2a\,(P+Q)}$$

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