

Cost and Reliability Interaction in Bridge Maintenance

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Abstract: This paper presents the relation between the cost of maintenance activities of a bridge and its reliability. Initially, based on fundamentals of reliability theory, a general methodology on bridge maintenance is introduced. Based on this maintenance strategy, the cost associated with different maintenance activities and its reliability increment is described. Having this kind of maintenance strategy is useful in bridge maintenance for a better understanding of the associated cost expenditures.

Keywords: Bridge maintenance, Reliability index, Failure probability, Cost

1. Introduction

In today's world, the quality of human life and economic progress of a particular country depend on the quantity, quality and efficiency of its infrastructures system [1]. However, many of the engineering structures in today's world are getting old and a large number of these is in need of maintenance, rehabilitation or replacement [2]. Among these, Highways and associated structures play an important role in the ground transportation systems and it constitutes a considerable investment in infrastructure and directly affects the productivity of the road transport industry of a country. Hence, condition estimation and proper maintenance of bridges have a widespread significance for continuous research.

The history of developing a maintenance strategy for bridges is probably a few decades old. However, in the process of bridge maintenance, a lack of a sufficient theoretical background has led to the non-availability of a widespread of accepted methods. As most of the current accepted strategies are based on visual inspections with varying time intervals [3], the uncertainty of the human factor is a disadvantage. In fact, human experience should always be backed with sufficient scientific knowledge. It is only then that any strategy gives an acceptable performance. However prevailing uncertainties in current procedures make a whole set of problems. The results of this inadequate knowledge in bridge maintenance are greater financial expenditure due to frequent maintenance activities, unforeseen bridge damages and accidental bridge failures. These losses in terms of financial, physical, reliability and other resources are not acceptable irrespective of the wealth of the country. Furthermore, when there is maintenance to be

carried out with different options, there is no defined method of selecting an option considered in relation to its cost and the increased service life of the bridge. But the cost consideration is very essential in bridge maintenance. The complexity of life prediction and bridge maintenance has hindered a lot of researchers in formulating a defined method of life prediction and a related cost optimized bridge maintenance strategy for bridges.

Thus, any future methodologies on condition estimation and maintenance should be based on results of field studies as well as scientific inferences to gain widespread acceptance among practicing engineers. Furthermore, since there are many uncertainties associated with current procedures, it is essential that any methodology proposed for the future be based on a probabilistic approach which caters for different uncertainties that have not been addressed by current ones. In this situation, reliability based methods can provide the rational approach to use scarce resources efficiently while maintaining a prescribed level of reliability of a structure throughout its designated service life.

While deterministic structural analyses provide output that is precise and exact, it ignores the uncertainties of the output. These uncertainties must be taken into account to assess the safety and the performance of the structure.

A reliability analysis can be used to explore the failure process and show which failure path is most likely to occur. Uncertainty modeling in

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structural evaluation can incorporate both the on-site inspection results and nondestructive evaluation data to provide a better estimation of the possibility of failure.

The reliability approach quantifies that risk in probabilistic terms by accounting for the randomness and correlation of all relevant variables in the analysis [4]. This probabilistic consideration of variables makes structural reliability more accurate over other conventional deterministic approaches. The concept of structural reliability has been used for over four decades in condition estimation of structures including bridges [5]. However, the data obtained from these research studies have not been incorporated into the bridge management databases in an effective way. Hence, there is an urgent need to combine reliability based research studies with condition estimation and maintenance of bridges to optimize resources expenditures in bridge management [6].

In Sri Lanka, an expansion and development of highways happened in the 1960's and 1970's. Now, most of these structures, at least 30 years old, need more attention than they needed in the original state to obtain service for more years to come. The condition of a bridge deteriorates due to two major effects; load increment over time and the aging effects. In a Sri Lankan context, bridge infrastructure is not subjected to heavy loads as in most of the developed countries. However, the aging effects resulting from degradation processes such as corrosion should not be unaccounted. Continuous actions of such degradation processes have created some of the worst problems at some bridge sites in the national bridge network of Sri Lanka. The ultimate impact of these processes is the complete collapse of bridges. A well known catastrophe is the collapse of Paragastota bridge in Bandaragama in the year 2000. Had proper inspection and maintenance been carried out on that bridge, the collapse could have been avoided. There are maintenance activities varying in different degrees to reduce the effects of these degradation processes on bridge infrastructures. However, the constraint for these activities is their related high costs. Hence, there is a need to study the relation between cost and reliability of bridges in terms of bridge maintenance.

Sri Lanka being a developing country and the government controlled Road Development Authority (RDA) being the organization responsible for national bridge network, a large amount of public money has to be spent on the road sector every year. From this amount, a significant amount of money goes on bridge maintenance. In this context, a careful understanding of cost and reliability interaction will drastically reduce resource expenditure on bridge maintenance.

The central issue in bridge maintenance is, how effective a particular maintenance activity in terms of the money spent is. Such a consideration is of extreme importance when there are a number of options of maintenance activities. It is normal that maintenance activities with a higher reliability increment have to be done at higher costs. An increase in reliability should be looked at with the intended increase in service life of the bridge in order to understand the effectiveness achieved at the higher cost [7,8].

2. Methodology

2.1 Development of Reliability Expressions

For a simple structural member selected at random from a population with a known distribution function F_R of ultimate strength R in some specified mode of failure, the probability of failure, P_f , under the action of a single known load effect S is as follows,

$$P_f = P(R - S \leq 0) = F_R(s) = P(R/S \leq 1) \quad \dots (1)$$

where R and S represent resistance and load variables and r and s denote values that those variables can have. In general, if the load effect S is a random variable, with distribution function, F_s , Equation 1 can be replaced by

$$P_f = P(R - S \leq 0) = \int_{-\infty}^{+\infty} F_R(x) f_s(x) dx \quad \dots (2)$$

where $F_R(x)$ is the distribution function for resistance variable and $f_s(x)$ is the density function of the load variable. Under the condition that R and S are statistically independent, Equation 2 can be best understood by plotting the density functions of R and S denoted by $f_R(r)$ and $f_s(s)$ respectively. It should be noted that r and s must necessarily have the



same dimensions e.g. loads and load-carrying capacities (e.g. failure strength), or bending moments and flexural strength etc.

Equation 2 gives the total probability of failure P_f as the product of the probabilities of two independent events, summed over all possible occurrences; namely the probability P_1 where S lies in the range from x to $x+dx$ and the probability P_2 where R is less than or equal to x . It is clear that

$$P_1 = f_s(x)dx \quad (3)$$

$$\text{and } P_2 = F_R(x) \quad (4)$$

Under these conditions the *reliability*, $\mathfrak{R}$, that is the probability that the structure will survive when the load is applied, is given by

$$\mathfrak{R} = 1 - P_f = 1 - \int_{-\infty}^{+\infty} F_R(x) f_s(x) dx \quad (5)$$

From the consideration of symmetry, it can be seen that the reliability may also be expressed as

$$\mathfrak{R} = 1 - P_f = 1 - \int_{-\infty}^{+\infty} (1 - F_S(x)) f_R(x) dx \quad (6)$$

where $F_S(x)$ represent the distribution function of load variable and $f_R(x)$ represents the density function of the resistance variable.

For the general case, close-form solutions do not exist for the integrals in Equations 5 and 6, but these two equations are very important because they represent the mathematical background to the reliability analysis. There are, however, a number of special simplified cases, and with the use of these, failure probabilities of structures can also be calculated.

2.2 Simplification to Fundamental Case

(R and S are independent normally distributed variables)

The condition that R and S are independent normally distributed variables is a simplified approximation to the reliability analysis. Failure probability P_f can be found only if variables are normally distributed. In case of non normal variables, these have to be converted to equivalent normal distributed variables and have to follow the procedure as mentioned below.

$$P_f = P(M \leq 0) \quad (7)$$

$$\text{where } M = R - S \quad (8)$$

$$\text{Thus } \mu_M = \mu_R - \mu_S \quad (9)$$

$$\text{and } \sigma_M^2 = \text{Var}[M] = \sigma_R^2 + \sigma_S^2 \quad (10)$$

$$\text{giving } \sigma_M = \sqrt{(\sigma_R^2 + \sigma_S^2)} \quad (11)$$

where μ_M , μ_R and μ_S are the probabilistic mean values of safety margin, resistance, and load respectively. Similarly σ_M , σ_R and σ_S are the standard deviations of safety margin, resistance and load respectively. $\text{Var}[M]$ is the variance of the safety margin. Since R and S are normally distributed and M is a linear function of R and S , safety margin M is also normally distributed and the quantity $(M - \mu_M) / \sigma_M$ is the unit standard normal variable of M . According to Equation 3.7, failure probability can be found when M is less than or equal to zero, thus, it would give

$$P_f = \phi\left(\frac{0 - \mu_M}{\sigma_M}\right) = \phi\left(\frac{\mu_S - \mu_R}{\sqrt{(\sigma_S^2 + \sigma_R^2)}}\right) \quad (12)$$

where ϕ is the standard normal distribution function and μ_S , μ_R , σ_S and σ_R are the probabilistic mean values and standard deviations of random variables R and S .

The reliability index β may now be defined as the ratio μ_M / σ_M or the number of standard deviations by which μ_M exceeds zero as shown in Figure 1. Here m represents a general value that M can take.

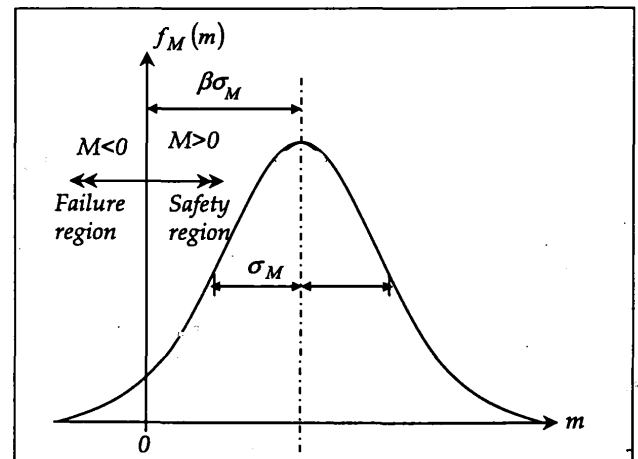


Figure 1: Illustration of the reliability index (β)

$$\text{Hence } P_f = \phi - \frac{\mu_M}{\sigma_M} = \phi(-\beta) \quad (13)$$

$$\text{And } \beta = -\phi^{-1}(P_f) \quad (14)$$

2.3 Graphical Representation of Reliability Index

The reliability index β is defined by Equation 14 for the fundamental case in Equation 8. For the general linear safety margin case it can be obtained by a simple geometrical interpretation. Hasofer and Lind proposed this graphical representation method of reliability index in 1974 and it is sometimes referred to as Hasofer and Lind reliability index.

Consider the fundamental case with independent basic variables R and S and the safety margin $M = R - S$. Let the mean values be μ_R and μ_S and the standard deviations be σ_R and σ_S . Introduce normalized random variables such that,

$$R' = \frac{R - \mu_R}{\sigma_R}, \quad S' = \frac{S - \mu_S}{\sigma_S} \quad (15)$$

Then the failure surface $f(r, s) = r - s = 0$ will be transformed into a straight line in the normalized (r', s') coordinate system as shown in Figure 3. Lower case letters (r', s') represent the values that variables R' and S' can take. By substituting Equation 14 to Equation 8, the failure surface in the (r', s') coordinate system can be obtained, and it is given by,

$$\sigma_R r' - \sigma_S s' + \mu_R - \mu_S = 0 \quad (16)$$

The shortest distance from the origin to this linear failure surface is equal to

$$\frac{\sigma_R \times (0) - \sigma_S \times (0) + (\mu_R - \mu_S)}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \beta \quad (17)$$

Therefore, an alternative geometrical definition of the reliability index β is the shortest distance from the origin to the linear failure surface as shown in Figure 2. This geometrical interpretation is shown here for a linear safety margin with only two basic variables, but it can be extended easily to a linear safety margin with n basic variables.

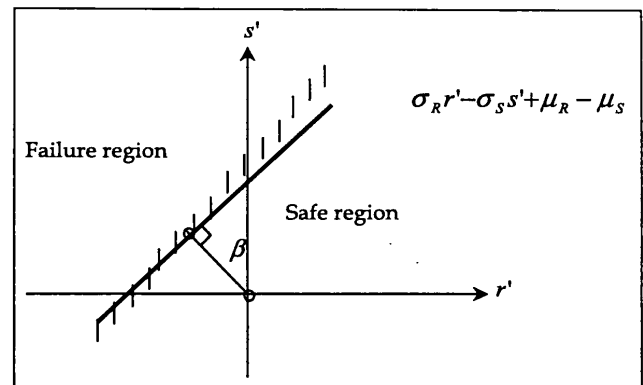


Figure 2: Graphical representation of reliability index

Proposed Reliability Maintenance Strategy

Studies have been done on critical failure criteria for different types of bridges. The inspection and maintenance strategy for different bridges can be derived from the reliability relationships introduced in the referenced literature [9]. In fact, the initial reliability index of a bridge is on the decrease as the bridge is exposed to traffic and to nature.

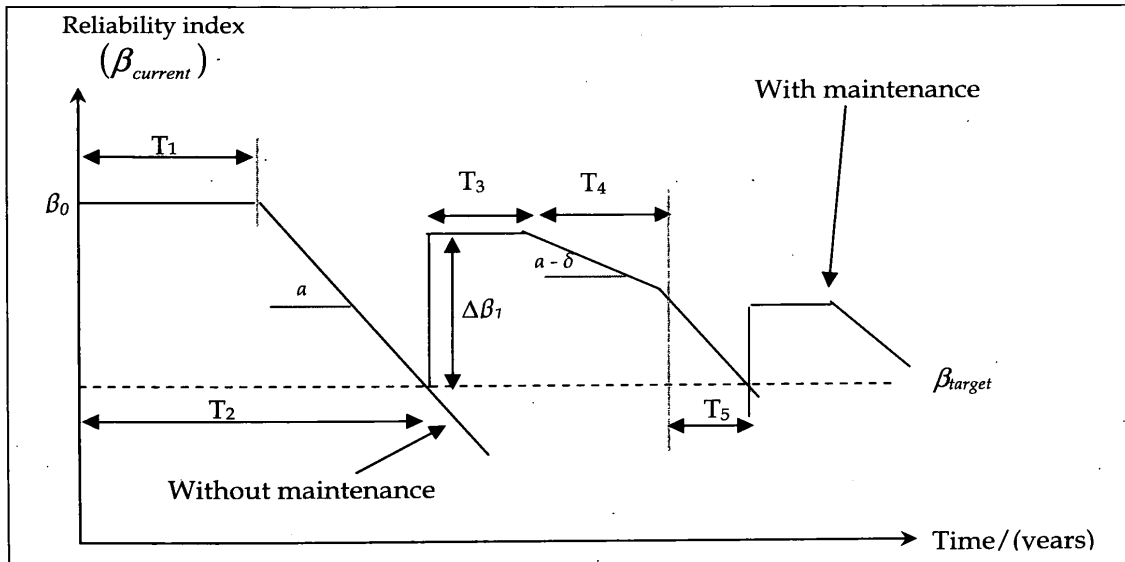


Figure 3: Graphical presentation of the maintenance requirement of a bridge



As illustrated in Fig. 3, the reliability index goes down with time. Initially, the value of the reliability index (β_0) after construction is a high value (> 6.0) and for a time period T_1 it remains the same. This is mainly because there is some time needed to initiate the degradation processes. After time T_1 , the reliability index reduces with the increase of time. During this period, the important aspect of study is the rate reduction of the reliability index (α). The value of α is dependent on the how the degradation process has affected the particular bridge. For example, a steel truss bridge located on a coastal environment has a higher α than a steel bridge located on land under the condition that, other factors such as load effects being the same the salt environment triggers the corrosion on steel. When the reliability index reaches target value, "Target Reliability Index" (β_{target}), the maintenance activity should be done. Maintenance activity such as replacing a heavily corroded girder in a steel girder concrete girder bridge. Such Essential Maintenance (EM) will increase the reliability index by an amount $\Delta\beta$. The value of increased $\Delta\beta$ should always accompany cost. Hence, it is generally essential to optimize increase of reliability index with cost (C). This increase of reliability index keeps constant for a period of time T_3 . After that, reliability index goes down with a reduced rate of degradation of $\alpha - \delta$. δ is called the reduction of deterioration rate due to maintenance activity. Its effect continues for a time period T_4 . Then, degradation rate becomes α again. When it reaches to β_{target} , maintenance should be done again.

2.4 Target Reliability Index

Many studies have been carried out for development of existing reliability design codes. The American Association of State and Highway Transportation Officials guide specification for steel bridges (AASHTO 1990) bases the remaining life of steel bridges on the use of a reliability oriented approach. In this guide, two levels of risk are considered. For a redundant bridge, a reliability index of 2.0 is used, and for a nonredundant bridge, a reliability index of 3.0 is used. However, more research should be carried out to find out target reliability index to suit a Sri Lankan context.

3. Case Study on a masonry arch bridge

3.1 Development of a Reliability Model for Masonry Arch Bridges

During the time most of the masonry arch bridges were constructed, the original designers had no idea about what kind of loading these arch bridges would have to sustain in future. In particular, they had no reason to predict present loading as they were not exposed to high traffic density that is prevailing at present.

With the heavy traffic both in magnitude and the number at present, it is very much essential to have some kind of evaluation of these masonry arch bridges on the basis of load carrying capacity that these could bear. Therefore, in the reliability analysis of masonry arch bridges, failure criterion is derived using the loads exerted on the bridge.

The proposed reliability expression must have a term for load capacity that simulates the strength variable and it must also have a term that expresses the current loading. With the selection of Provisional Axle Load (PAL) as the strength variable and Actual Axle Load (AAL) as the load variable, the reliability expression or the safety margin concerning the masonry arch bridge can be built. The PAL is the maximum permissible load on the selected masonry arch bridge and the AAL is the load that is applied on the bridge at a selected time. Hence, the reliability model of the masonry arch bridges can be proposed as,

$$M = PAL - AAL$$

where M is the safety margin, PAL is the Provisional Axle Load in kN and AAL is the Actual Axle Load in kN .

Both axle loads are not deterministic quantities in the true sense. In this regard, those are assumed to behave as random variables with some probabilistic parameters. Therefore, those are modeled as variables having some probabilistic distributions.

3.2 Selection of a Masonry Arch Bridge

Taking into consideration all the above facts, a single spanned stone masonry arch bridge (71/1) situated close to Hatton town in A7 road was selected for the case study with the help of the Road Development Authority. This bridge was

constructed in 1918 and it has been in operation ever since. It has the following geometric details,

Bridge length = 14 m,
Clear span (L) = 8.8 m,
Thickness of the barrel (d) = 0.55 m,

Height of the compacted fill from the crest of the barrel (h) = 1.50 m.

Those geometric details are as shown in Figure 4 and a photograph of the selected bridge is shown in Figure 5.

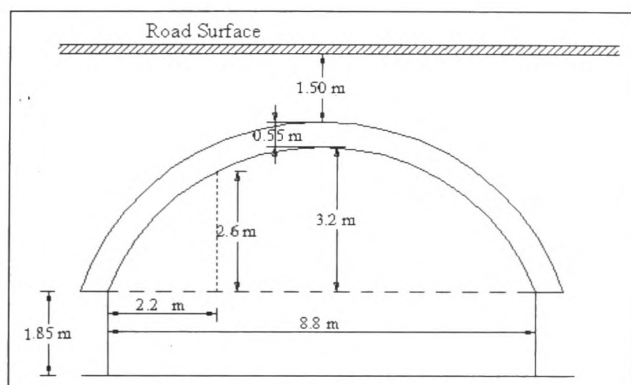


Figure 4: Geometric details of the selected arch bridge



Figure 5: A view of the selected arch bridge at 71/1 along A7 road

The current state of the bridge is visually satisfactory except for a few signs of water seepage through masonry blocks in the inner barrel of the arch bridge as shown in Figure 6.



Figure 6: Seepage through blocks

If this is continued, blocks may become loose and would tend to behave as individual blocks. In fact, this was one of the main reasons for the selection of this masonry arch bridge as a case study. More importantly, there is a significant increment of the traffic flow over this particular bridge since the upgrading of road A7 and this could significantly reduce the life of the bridge. Hence, proper attention should be paid to the current condition of the bridge.

3.3 Probabilistic Parameters of Actual Axle Load (AAL) and Provisional Axle Load (PAL)

Axle load measurements over the bridge were carried out and it was found that nearly 2570 vehicles had passed over this bridge under consideration. From this population, a random sample of 293 vehicles was selected as a sample population. Axle loads were measured for this sample population.

From Axle load measurements, it is found that AAL has a mean of 50.7 kN and standard deviation of 29.03 kN. From the central limit theorem, it can be concluded that AAL measurements obey normal distribution even if the parent distribution is not normal.

In the determination of PAL, the modified MEXE method was used. Initial value of PAL was measured as 184.03 kN. With the geometric details five adjustment factors in the modified MEXE method were calculated. Using those, the modified PAL was calculated as 233.91 kN. In fact, this value itself has an uncertainty. Hence, to counter this uncertainty, three values of Coefficient of Variation (COV) were used.

3.4 Calculation of Reliability Index and Failure Probability for the Selected Bridge

Having found the probabilistic parameters of the PAL and AAL, it is possible to calculate the reliability index and the failure probability. Tabulated values of probabilistic parameters of PAL and AAL are given in Table 1.

Table 1: Probabilistic parameters of PAL and AAL

Variable	Mean/(kN)	Standard Deviation/(kN)		
		COV = 0	COV = 0.1	COV = 0.2
PAL	233.91	0	23.39	46.78
AAL	50.7	29.03		



3.5 Results

Having known the probabilistic parameters of the PAL and AAL, the reliability index and the failure probability can be obtained. These values are given in Table 2.

Table 2: Reliability index and failure probabilities for the selected bridge.

Case	Reliability Index / (β)	Failure Probability / (P_f)
Case I: when COV of PAL = 0	6.31	1.5×10^{-10}
Case II: when COV of PAL = 0.1	4.91	4.8×10^{-7}
Case III: when COV of PAL = 0.2	3.32	4.5×10^{-4}

From the results shown in Table 2, for first and second cases, the failure probabilities are less than 10^{-5} . The third case represents the extreme possibility and it shows a failure probability of more than 10^{-5} . Since the target failure probability is 10^{-5} , as a conclusion, it can be expressed that the selected bridge is currently in a safe condition but fair attention is needed in future regarding the axle loads on the bridge.

Detailed monitoring of the places where seepage is observed in the inner surface of the arch barrel should also be carried out. The number of such seepage paths should be counted periodically and if possible their average sizes should be measured. Any increase of such seepage paths and path sizes should be thoroughly investigated.

4. Cost Consideration in bridge maintenance

4.1 Cost and reliability index interaction

In the maintenance management of bridges, there are two improvements happening in terms of the reliability of the bridge after maintenance is carried out; increase of reliability index and reduction of the rate of deterioration of reliability index. Hence, simplified cost function can be written as,

$$C(t) = f(\beta(t), \Delta\beta(t), \delta(t)) \quad (18)$$

$C(t)$ is the cost function and $\beta(t)$ is the reliability index at time t and $\Delta\beta(t)$ is the increase of reliability index and is the reduction of the deterioration rate of reliability index. In this context, cost related to inspection stage has not been taken into consideration. The value of $\beta(t)$ is dependent on cost function in such a way that lower values of $\beta(t)$ require higher values for C .

As the first case, interaction between the cost and reliability index is studied. This consideration of reliability index with the cost is of importance in setting a target reliability index. The corresponding relationship between cost and reliability index can be set as follows,

$$C_{\beta(t)} = C_0 + p(\beta(t))^q \quad (19)$$

Where $C_{\beta(t)}$ is the cost associated with current reliability index; C_0 is the fixed cost irrespective of current reliability index value; $\beta(t)$ is the current reliability index. p and q are the model parameters.

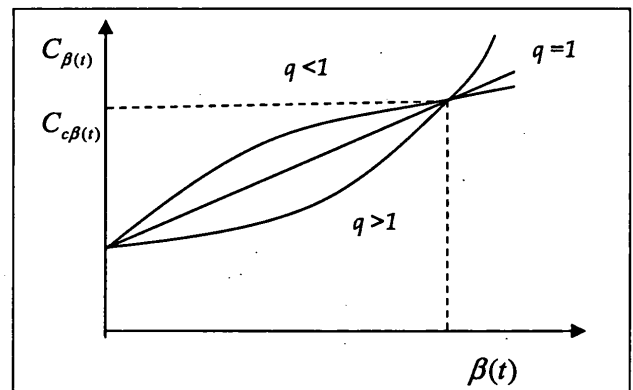


Figure 7: variation of cost and reliability of a bridge.

By substituting the values at common point as the calibration point, the parameter p can be found.

$$p = (C_c - C_0) / \beta_c \quad (20)$$

4.2 Cost and increase of reliability index interaction

The interaction between the cost and the reliability index is important to study effective bridge maintenance. From all three parameters that affect the bridge reliability after maintenance, this is the most important interaction. In general terms, increase of reliability $\Delta\beta(t)$ index is higher for Essential

Maintenance (EM) than in the case of Preventive Maintenance (PM). Hence the cost associated with Essential maintenance is higher than preventive maintenance. Cost function for this interaction can be represented as follows,

For Essential maintenance,

$$C_{\Delta\beta(t)} = C_{0\Delta\beta} + l(\Delta\beta(t))^m \quad (21)$$

For Preventive maintenance,

$$C_{\Delta\beta(t)} = l(\Delta\beta(t))^m \quad (22)$$

Where $C_{\Delta\beta(t)}$ is the cost associated with increase of the reliability index at the time t . $C_{0\Delta\beta}$ is the cost component independent of increase of the reliability index. $\Delta\beta(t)$ is the increase of the reliability index at time t . P and q are the cost parameters. In the case of preventive maintenance, the fixed cost is small compared to the latter component and hence cost function in the case of preventive maintenance is represented by variable cost associated with reliability increase. The parameter value l for Essential maintenance is higher than that of preventive maintenance resulting in higher value for the cost function in the case of EM than in PM.

4.3 Cost and reduction of deterioration rate of reliability index

Due to the maintenance activity carried out, it inserts a reduction of deterioration of reliability index $\delta(t)$ on the deterioration rate of reliability index $\alpha(t)$ at the time t . Normally, for effect of $\delta(t)$ on the bridge maintenance, EM makes a significant contribution whereas contribution from PM is not significant. The relationship between $\delta(t)$ and the cost C is such that for higher values, $\delta(t)$ the associated cost is higher. Hence cost function can be illustrated as follows,

$$C_{\delta(t)} = x(\delta(t))^y \quad (23)$$

Where $C_{\delta(t)}$ is the cost associated with the reduction of the deterioration rate of the reliability index. X and y are the cost parameters associated with this case.

4.4 Expression for the Total cost of maintenance

Total cost for maintenance activity either PM or EM can be developed. In this study, it has assumed that component costs are independent

with no correlation between them. Hence, total cost of maintenance for maintenance activity of EM can be expressed as follows,

$$C = C_{\beta(t)} + C_{\Delta\beta(t)} + C_{\delta(t)} + C_{\text{Other}} \quad (24)$$

Where C is the total cost and C_{Other} is the fixed additional cost such as inspection cost etc. Thus the expression can be expanded from relationships obtained as above.

$$C = C_0 + p(\beta(t))^q + C_{0\Delta\beta} + l(\Delta\beta(t))^m + (\delta(t))^y + C_{\text{Other}} \quad (25)$$

In a similar way, the cost associated with PM can be expressed as follows,

$$C = C_0 + p(\beta(t))^q + l(\Delta\beta(t))^m + C_{\text{Other}} \quad (26)$$

Equations 25 and 26 can be read as an optimization problem and thereby minimization of the cost can be obtained while maximizing $\Delta\beta(t)$ and $\delta(t)$. That will give the optimum maintenance to the bridge.

5. Conclusion

This study was focused on how the reliability index and cost are dependent on each other for different maintenance activities such as PM and EM. It also illustrated when maintenance is required and how a maintenance activity can affect the bridge reliability. With this theoretical understanding as illustrated better bridge maintenance can be carried out in the national road network of Sri Lanka.

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