LUMPING SCALE OF WATERSHEDS IN RELATION TO RUN-OFF PHENOMENON

by

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Abstract

Runoff analysis of a catchment could be made simple and easy by considering the whole catchment at once and taking average values for parameters such as slope, gradient etc. However the degree of accuracy may be low. On the other hand it could also be considered in detail by disintegrating the catchment and taking small sub catchments into consideration which may bring in greater accuracy. The least degree to which the catchment could be disintegrated maintaining appreciable accuracy of the resulting runoff calculations is known as the lumping scale of the catchment and larger the lumping (lower the disintegration) the easy the calculations. (Lumping is the consideration and putting to gether of many elements as one.) However the elements that affect lumping most such as slope, gradient, must be clearly identified and the threshhold to which lumping must be curtailed be known.

In this research, the catchment is considered as a combination of one, two, three or four slopes. Thereafter analyzing using a simple model developed herein, the elements of the catchment that are important in performing lumping in relation to runoff have been clarified. It is found that there are catchments which geographically make lumping easy such as the fan shaped ones, as well as catchments that make lumping difficult such as the ones where the standard deviation of the slope lengths become high and the catchment involves extremely long slopes.

A theoretical example as well as a case study of a real catchment is presented to make clear that effective lumping is possible using three or four representative slopes for the complex catchment or a single average representative slope for the simple catchment. It is also shown that assuming average values such as average slope length and average gradient, it is possible to do a good analysis for small mountainous catchments. It is also found that while the slope length acts linearly on the run-off parameters the run-off duration could be represented as an exponential function of the gradient.

1.0 Introduction

In this paper a simple and effective method to analyze the rainfall run-off phenomenon is investigated. One problem that comes up in the process of simplification of analysis is the estimation of the lumping scale of the catchment. If the lumping scale of the catchment could be taken large as possible but conserving the physical meaning of the run-off phenomenon and maintaining accuracy, then it is appropriate to use large scale lumping [Ishihara and Kobatake 1978]. However there exists many unexplained points in relation to the lumping scale of a catchment, especially the hillside catchments. Takasao [1978] as well as Takagi [1978] have shown the benefits of large scale lumping in comparatively large catchments where ground water flow was present. Here the effect of the ground water flow evening out in large scale lumping has been shown. However, they do not lead to the accurate identification and estimation of the elements that are important to perform lumping on a hillside. Hirano [1980] has shown that it becomes increasingly difficult to identify and estimate the elements that affect runoff if the slope length and gradient are considered to act simultaneously.

Therefore, taking into consideration the strength and weakness of the above models in this paper, a rain water run-off model is developed and applied, to analyze the lumping of mountainous slope, especially of small mountainous catchments. The findings of this study would be useful when one is attempting to accurately calculate the discharge of a catchment. As then decisions have to be made with regard to the degree to which the catchment must be disintegrated

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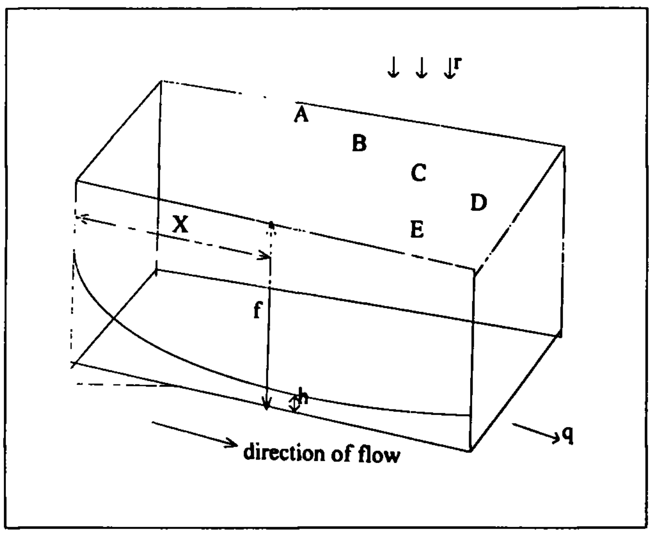
(how many number of elements must be considered or, the lumping scale of each of the factors being considered namely, gradient of the elements, slope lengths of the elements).

2.0 Run off model

2.1 Run off equation

A simple model for the analysis of run-off generated due to rainfall on a unit slope is presented. The model is developed using the permeability ($K_A < K_B < K_C < K_D$) of sub sections A, B, C and D. These sub sections are introduced to treat saturated/unsaturated flow in soil. Since the real soil stratum contains varying degrees of saturation resulting in varying flow rates, here these conceptual sections have been assumed. Here it is assumed that the water in each section is considered to flow according to Darcy's law, and the water between each section is balanced by capillary force.

Using the surface layer thickness d and the coefficient of permeability of a general section B as KB, and non dimensional quantities as in equation (1), the non dimensional equations of flow could be expressed as in eqs. (2) to (12). Here, we suppose that the water of section A is held firmly and will not flow with high capillary force [Mishima et al 1990].



direction of flow

$$X = \frac{X}{d} \qquad T = \frac{k_A t}{d} \qquad Kj = \frac{k_j}{k_B}$$

$$R = \frac{r}{k_B} \qquad L = \frac{l}{d} \qquad F = \frac{f}{d}$$

$$Hj = \frac{h_j}{d} \qquad Qj = \frac{q_j}{k_B d}$$

Here, f is the sectional depth, h is the height of water, t is time, x is distance from the upstream end, d is the surface layer thickness, 0 is the angle of the slope, r is the intensity of effective rainfall, q is the corresponding discharge of each section, k is the permeability and n relates to the porosity of each section. The capital letter (upper case) subscript indicates the section, the small letter subscript u, d, m and j indicate the upstream, the downstream, the value related to capillary water and the representative letter for the sub section.

Applying the equations for the conservation of mass (total input equals total out put) and using the non dimensionalized values of equation (1),

$$n_B \frac{\partial F}{\partial T} + \frac{\partial Q_B}{\partial X} = R\cos\theta \qquad (2)$$

$$Q_B = n_B F \cos\theta \left(\tan\theta + \frac{\partial F}{\partial X} \right) \qquad (3)$$

$X_{Cu} \leqq X < X_{Du}$:

$$n_C \frac{\partial H_C}{\partial T} + \frac{\partial Q_C}{\partial X} = \frac{(R\cos\theta - n_B \sin\theta)}{n_C(1 - X_{Cu})} \qquad (4)$$

$$Q_C = n_C H_C \cos\theta \left(\tan\theta - \frac{\partial H_C}{\partial X} \right) \qquad (5)$$

$X_{Du} \leqq X < 1$:

$$(n_C + n_D) \frac{\partial H_D}{\partial T} + \frac{\partial (Q_C + Q_D)}{\partial X} = \frac{R\cos\theta - n_B \sin\theta)}{(n_C + n_D)(1 - X_{Du})} \qquad (6)$$

$$Q_C = n_C K_C H_C \cos\theta \left(\tan\theta - \frac{\partial H_D}{\partial X} \right) \qquad (7)$$

$$Q_D = n_D K_D H_D \cos\theta \left(\tan\theta - \frac{\partial H_D}{\partial X} \right) \qquad (8)$$

The discharge at the downstream of sections B. C and D. Q_{Bd}. Q_{Cd} and Q_{Dd} are.

$F_d < (1 - H_{Bm})$:

$$Q_{Bd} = n_B F_d \cos\theta \left(\tan\theta + \frac{\partial F_d}{\partial X} \right) \qquad (9)$$

$(1 - H_{Bm}) \leqq F_d$:

$$Q_{Bd} = n_B \sin\theta \qquad (10)$$

$$Q_{Cd} = n_C K_C H_{Cm} \cos\theta \left(\tan\theta - \frac{\partial H_C}{\partial X} \right) \qquad (11)$$

$$Q_{Dd} = n_D K_D H_{Dm} \cos\theta \left(\tan\theta - \frac{\partial H_D}{\partial X} \right) \qquad (12)$$

2.2 Values for permeable characteristics

The value for the permeable characteristics of the section of the catchment that is being considered is obtained using an experimental slope (hill) made of gravel which is the representative soil of the area. The mean diameter of the soil particles is 1.0mm. Slope length = 14m, surface layer thickness d = 25cm, gradient tan 0 = 1/6, porosity is 41%.

To this slope an experimental artificial rain is used and the experimental hydrograph is observed. Thereby, the coefficients of the previously mentioned equations are calculated and they are as follows.

kC = 2.4, kD = 22, nB = 8.6%, nC = 10.9%, nD = 9.7%,

HBM = 0.75, HCM = 0.36, HDM = 0.09

The calculated and experimental hydrographs are shown in Fig, 1. As the two match well these values are taken as the typical values of the permeability characteristics of the unit slope.

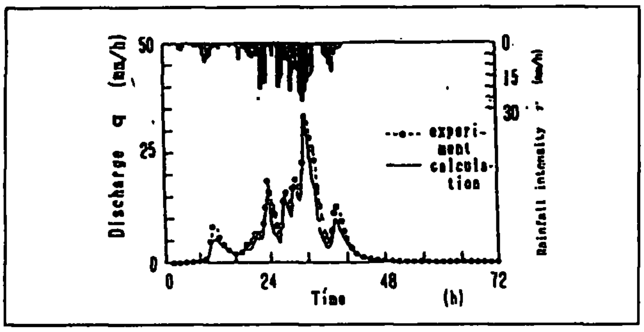


Fig 1: Comparison of the experimental Hydrographs

3.0 Typical rainfall and data

It is generally thought that a total rainfall quantity more than 30mm for a given rainfall will cause surface run-off [Kanamaru et al 1991]. Here as the flow within the layer is mainly being investigated and as uniform rainfall yields a smaller run-off, a rainfall intensity of r = 5mm/h and rainfall duration of t = 3 hours is considered. An observed rainfall is also applied. As for the method of calculation, a finite difference method incorporating the non dimensional values are used. The space increment x is taken as 1/20 and the time increment t is taken as 1/200 so that the calculated values will not diverge.

4.0 Theoretical analysis of the influence of the catchment elements

The representative geophysical factors of the slope that affect the rain water run-off would be gradient ten ø, slope length and surface layer thickness. However as understood from the calculations stated in section 2, as the effect of surface layer thickness is included in the slope length factor the surface layer thickness is taken as constant (d = 25 cm) and the effect of both the other elements are investigated.

4.1 The effect of the gradient

To investigate the effect of the gradient, a uniform slope length 1 = 15 cm and gradients tan 0 = 1/12, 1/4 and 1/2 are selected. As seen from Fig. 2, if the gradient increases, the change of the run-off becomes rapid and the run-off concentrates more easily. Also as the gradient decreases the run-off attenuates, the discharge tends to have fewer peaks and a long duration is necessary to reach a steady state. Even when the slope length is increased to 30m and 45m, the same tendencies are observed.

As could be seen from Hirano's [1980] research the run-off duration tq(h) for 1 = 15m could be approximated to the following equation,

$$t_q = 61.7\, e^{-2.16 \tan 6} \qquad (13)$$

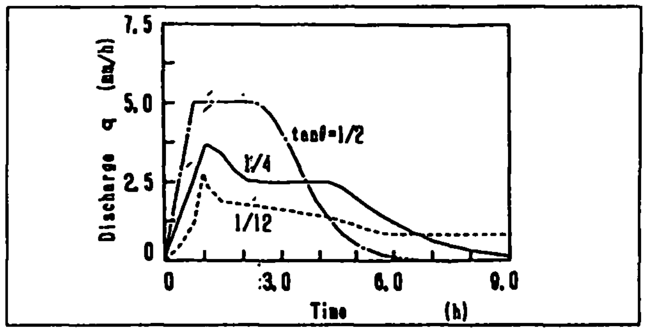


Fig - 2 Variation of Discharge with change igradient

4.2 The effect of the slope length

Fig. 3 indicates the graphs for tan 0 = 1/2 and slope lengths 1 = 15m, 30m and 45m. It can be seen that the run-off phenomenon becomes dull and the duration of run-off becomes long as the slope length increases. The run-off duration is proportional to the slope length and the duration of each slope matches well with the duration calculated from equation (13).

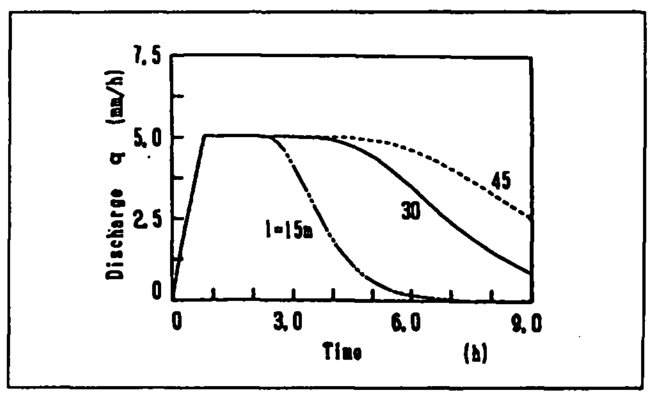


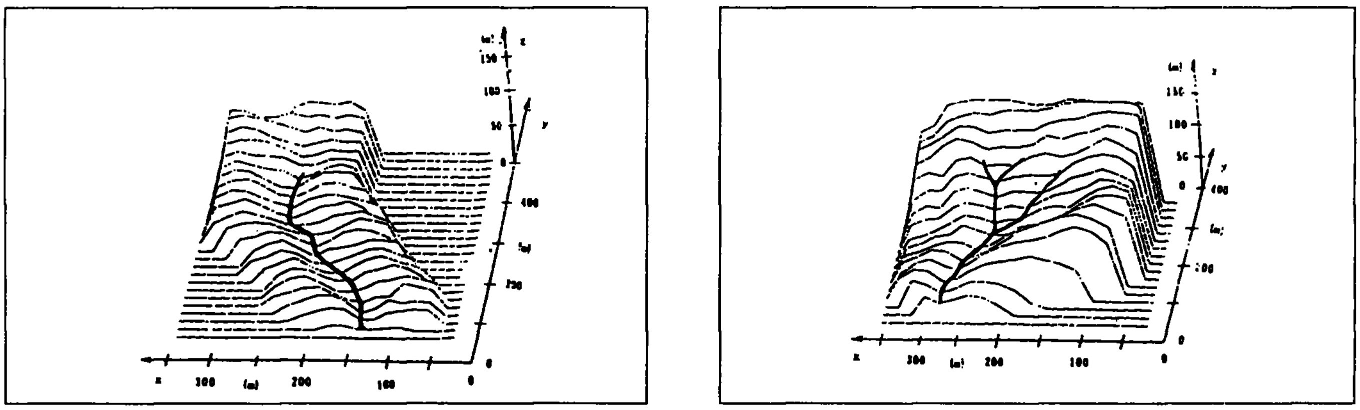
Fig - 3 Variation of discharge with change in slope length

5.0 Investigation of a real catchment

On the basis of the previously stated basic results, first a typical small mountainous catchment is analyzed and then lumping is investigated. Generally in the run-off of a real catchment, there is a time lag for the run-off water to enter the channel [Mishima and kanamaru 1991]. However when the catchment area is less than 0.2km2, the time lag of the catchment is less than 30 minutes [Mishima et al 1992]. As the time lag of the misasa catchment is about 10 minutes and is small considering the whole run-off duration, in this analysis of the misasa catcment the time lag is neglected.

5.1 Misasa catchment of Japan

The catchments (A, B) which are being considered are in the outer periphery of the northern part of Hiroshima city, and fall into the midstream of the Misasa river that is in the Ota river basin. The contour characteristics are shown in Fig. 4. The characteristics of the catchment are shown in Table 1. The soil quantities and type of both catchments are quite similar. However catchment B has a comparatively more complex configuration. Both catchments are quite well covered with pine trees. The surface layer thickness (the layer which relates to water flow) [Mishima et al 1993] is taken as 50 cm for catchment A and 75 cm for catchment B as indicated in Table 1. The thickness of fallen and dead leaf on the surface (thickness of dead leaf sediment) is taken as 10 to 15cm for both catchments as indicated in Table 1.



(a) Catchment A

(b) Catchment B

Fig - 4 Isometric view

Table -1 Catchment Characteristics

	Catchment A	Catchment B
Catchment area (ha)	4 5	4.3
Height above mean sea level (m)	325 - 410	343 - 490
Length of main channel (m)	361	207
Average slope length (m)	30	43
Average gradient	0.55	0.69
Soil type	Gravel	
Plantation	30 Years old pine and broad leaf type	
Thickness of dead leaf sediment (m)	0.1 - 0.15	
Surface layer thickness (m)	0.5	0.75

5.2 Results of the geographical analysis

Using a map of scale 1/2000, the slope length and gradient at every 10m is plotted This is shown in Fig. 5. It could be seen that, catchment A's factors are lumped together and more correlated while B's are scattered. Catchment A and catchment B are divided into 254 and 157 slopes respectively.

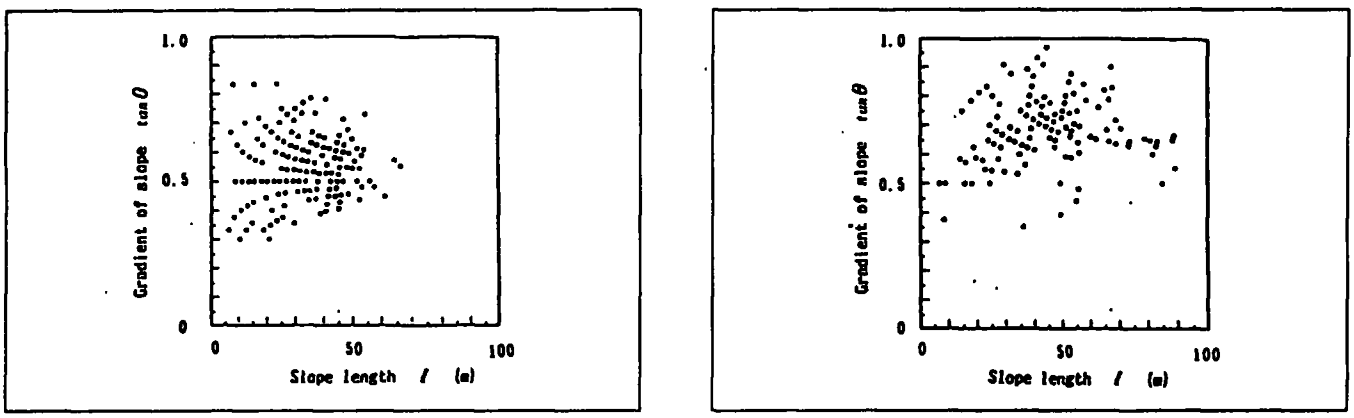
On the basis of the degree of influence of the slope element distribution and grouping, catchment A can be considered to have 4 types of unit slopes while catchment B can be considered to have 3 types of unit slopes as indicated in Table - 2.

5.3 Investigation of lumping

5.3.1 Investigation related to uniform rainfall

A catchment that has several slopes such as, catchment A (254) and B (157), have been analyzed in three different ways considering a rainfall intensity of 5mm/h and a duration of 3 hours. One method is considering each slope element (Fig. 6; AT and BT). AT and BT refer to total slope elements. Another method is where the slopes are considered to be totally lumped (this is called a simple average slope) and run-off is obtained. (Fig. 6 AS and BS). The run-off is also obtained considering the catchment to posses typical compound slopes (Fig 6; AC and BC). AC and BC refer to the compound slopes, 3 or 4 typical slopes only. They are as indicated in Table 2.

As seen from Fig 6, the differences are small in catchment B even if lumping has been done on large scale or small scale. It could be seen that lumping will yield accurate results in relation to the peak run-off as well as at the tail of run-off. Catchment B which is fan shaped yields more accurate results than catchment A the complex sloped catchment.



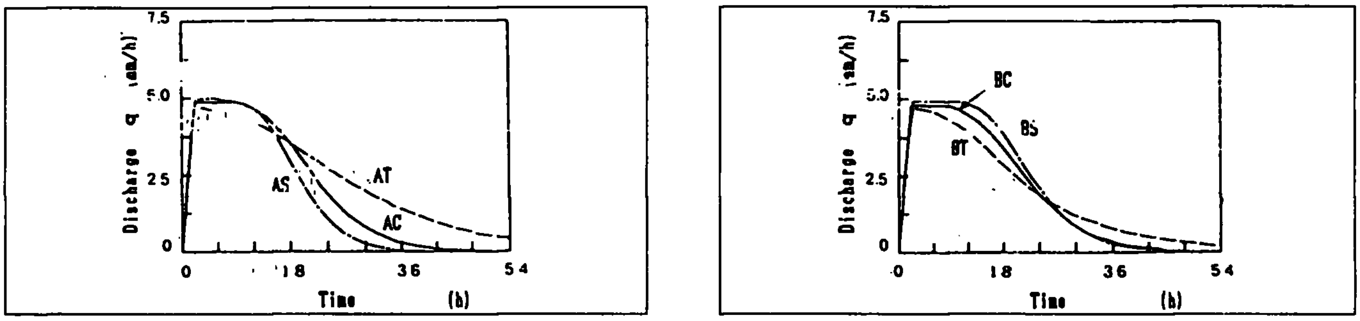
(a) Catchment A

(B) Catchment B

Fig - 5 Relationship between gradient tan0 and slope length l

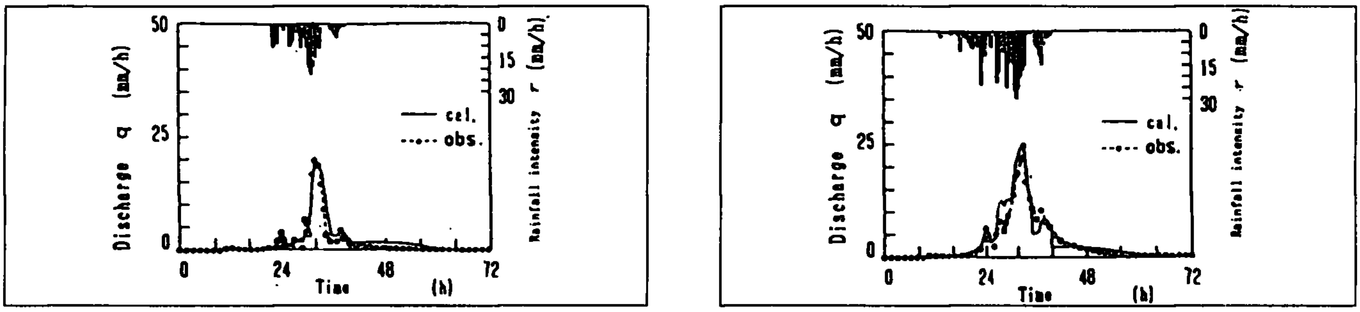
Table - 2 Characteristics of the typical slopes

	Catchment A				Catchment B		
Typical gradient (tan θ)	0.55	0.55	0.65	0.45	0.7	0.69	0.55
Typical slope length (m)	50	35	20	20	65	43	20



(a) Catchment A (b) Catchment B

Fig - 6 Comparison of the hydrographs for the typical compound slopes and the lumped slopes of catchments A and B



(a) Catchment A (b) Catchment B

Fig - 7 Comparison of the observred and calculated hydrographs of a typical compound slope.

5.3.2 Investigation related to distributed rainfall

After dragging the losses from the real rainfall occuring on a real catchment, the effective rainfall is found. This is applied to the compound slopes as well as the totally lumped catchment and the calculated hydrographs are compared with the real hydrographs.

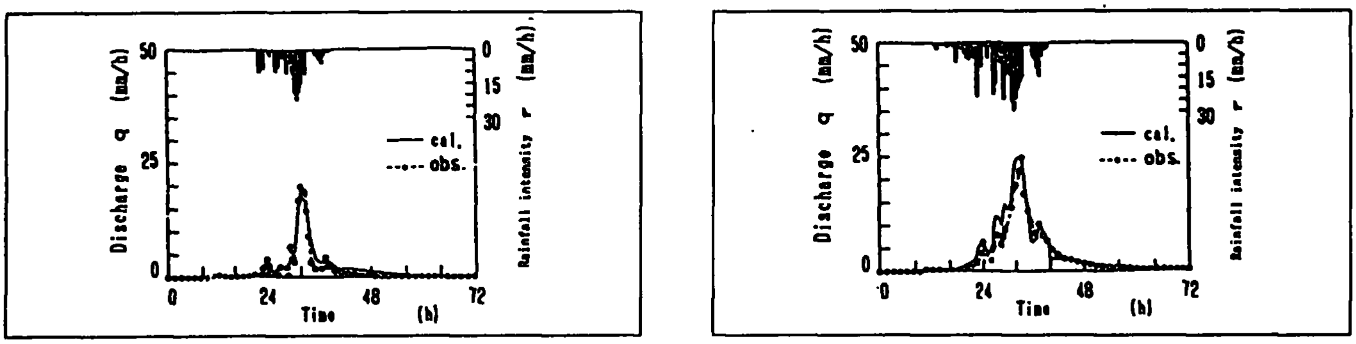
The data that is used is the data for the year 1983 September 27 to 29 at 30 minute intervals. The effective rainfall is calculated by the Horizontal Separation Method that deducts the uniform base flow discharge from the real discharge [Mishima et al 1994]. The run-off ratio is 43% for catchment A and 89% for catchment B. The base flow of catchment A and B is 0.12mm/h and 0.044mm/h respectively.

It could be seen from Fig. 7 that the calculations show high degree of simulation and this clarifies the effectiveness of lumping.

Fig. 8 indicates the comparison of real values with once calculated assuming one average value for the slope length and one average value for the gradient (simple slope). In this case too, a satisfactory degree of accuracy has been achieved. Fig. 9 indicates the inter relation between the discharge qg of the typical compound slope (considering as 3 or 4 slopes, namely taking 3 or 4 average values for gradient and slope length) and the discharge qs of the simple average slope (considering as one slope namely using one average value for gradient and one average value for slope length). For these catchments, the plot tends to a distribution around a 450 line on a logarithmic graph. Therefore it could be said that, if lumping is done taking good representative values for the factors related to the catchment, a sufficiently satisfactory result can be obtained.

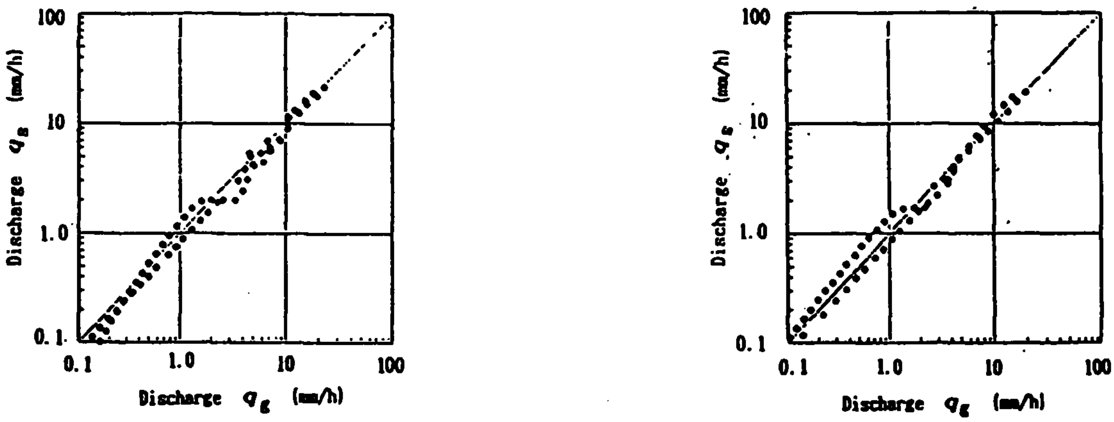
6.0 Conclusion

Taking into account the degree of unsaturated water flow, the lumping scale of a catchement is investigated with a simple and evident method. Here it is found that as the gradient increases, the change in run-off becomes rapid and the discharge tends to concentrate. It is also found that as the gradient decreases the peak run-off attenuates. The slope length acts linearly (has a linearly varying impact) on the run-off phenomena. It could be concluded that calculations resulting from lumping of catchments yields appreciably accurate values for the peak run-off and also at the tail of run-off (descending limb of the hydrograph). However,when considering the scale of lumping the more the slopes considered the more accurate the calculated values specially during the recession period. It is also clear that lumping is easier performed with greater accuracy for the fan shaped catchments than that for, catchments with high deviations of gradients as well as catchment with long slope lengths.



(a) Catchment A (b) Catchment B

Fig - 8 Comparison of the observed and calculated hydrographs of a simple average slope.



(a) Catchment A (b) Catchment B

Fig - 9 Correlation between the discharge qg of a typical compouned slope and discharge qs of a simple average slope

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