OPT M ZATION OF DA .Y PRODUCTION UNDER VARYING RESOURCES - AN EXAMPLE

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INTRODUCTION

This is a kind of problem where a production supervisor has to decide on a day's target of manufacturing different types of items so as to maximise profits.

A solution without the consideration of variations in the manufacturing constraints is of very little practical use. In this study variations due to time, labour and materials are considered in addition to sales forecasts and minimum requirements in the production of eight different types of concrete hume pipes.

Tables giving a day's target schedule are produced for the production supervisor's use whereby he reads the day's production rate corresponding to the available resources.

THE PROBLEM

A certain company manufactures eight different types of concrete hume pipes. It is desired to know how many of each type should be made each day, in order to maximize profits after satisfying all manufacturing and selling constraints.

Table 1 gives all details of pipes, including costs of manufacturing and selling prices. The minimum requirements and maximum sales forecasts are given in Table 2.

TABLE 1 – DETAILS OF PIPES

TYPE	PIPE DIA AND LENGTH	CONCRETE REQUIREMENT PER UNIT IN LBS.	LABOUR REQUIREMENT PER UNIT IN MANHOURS	DURATION TO MAKE ONE UNIT IN MINS.	COST IN TT $	SELLING PRICE IN TT $
	72"x36"	4700	7.00	125.00	380	585
B	60"x36"	3130	7.00	125.00	265	409
C	48"x36"	2610	4.75	30.00	255	398
D	36"x36"	1330	1.00	8.57	65	103
E	30"x36"	905	0.73	6.67	50	82.50
F	24"x36"	650	0.63	5.63	40	62
G	18"x36"	455	0.44	4.00	25	38.75
H	12"x48"	290	0.17	2.06	16	26.50

TABLE 2 – MINIMUM REQUIREMENTS AND MAXIMUM SALES FORECASTS PER DAY

TYPE	A	B	C	D	E	F	G	H
Minimum Requirement	2	2	4	8	6	14	28	14
Maximum Sales Forecasts	4	4	8	28	36	42	64	116

The minimum requirements are certain orders the company has to supply. The sales forecasts are based on previous sales patterns.

The pipes are concreted in different machines. However, when the machine for the type C is in production, machines for types, D, E, F, G and H have to stay idle. But if machine for type C stays idle then, all other machines for types D, E, F, G and H could be in production. Machines for types A and B could operate independently.

The Linear Programming Model

Decision Variables

The decision variables in this problem are the number of pipes, in each type, to be made each day.

Let

A	=	Number of pipes of type	A
B	=	- do -	B
C	=	- do -	C
D	=	- do -	D
E	=	- do -	E
F	=	- do -	F
G	=	- do -	G
H	=	- do -	H

Manufacturing Constraints

There are several manufacturing constraints that has to be considered.

1. Labour requirements

The total number of men the company is prepared to provide are 25. Each pipe requires a certain amount of manhours. The total requirement of manhours to make the pipes should be less than or equal to what is provided. The required manhours per unit are given in Table 1.

The company works on an 8-hour day shift. The efficiency is taken as 0.75.

$$\text{Available manhours per day} = 25 \times 8 \times 0.75 = 150$$

Therefore,

$$7A + 7B + 4.75C + D + 0.73E + 0.63F + 0.44G + 0.17H \leq 150$$

2. Concrete Requirements

The concrete is supplied by two mixing plants P and Q. Plant P has an output of 1800 lbs. of concrete per cycle and Plant Q has an output of 1200 lbs. of concrete per cycle. Each plant makes 7 cycles per hour.

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The production of concrete from Plant P per day is equal to :

$1800 \times 7 \times 8 \times 0.75 = 75{,}600$ lbs.

Production from Plant Q per day is equal to :

$1200 \times 7 \times 8 \times 0.75 = 50{,}400$ lbs.

The total concrete supply per day, from the two plants are equal to 126,000 lbs. The concrete requirements for all pipes should be less or equal to the supply. The amount of concrete required per unit is given in Table 1.

Therefore

$$4700A + 3130B + 2610C + 1330D + 905E + 650F + 455G + 290H \leqslant 126{,}000$$

3. Time Requirements

The total time available per day, in minutes, is equal to

$8 \times 0.75 \times 60 = 360$ minutes

Pipes A and B require 125 minutes per unit.

Therefore, $125A \leqslant 360$
and
$125B \leqslant 360$

Pipes C and D require 30 and 8.57 minutes per unit respectively.

However, when one is in production the other idles.

Therefore,

$30C + 8.57D \leqslant 360$

Similarly, the time constraints for the pipes E,F, G and H are

$30C + 6.67E \leqslant 360$
$30C + 5.63F \leqslant 360$
$30C + 4.00G \leqslant 360$
and $30C + 2.06H \leqslant 360$

Sales Constraints

The company should make sure that it is possible to sell whatever they make, in the market. Therefore, they should not exceed the sales forecasts. The forecasts per day are given in Table 2. The sales constraints are :

$A \leqslant 4$	$E \leqslant 36$
$B \leqslant 4$	$F \leqslant 42$
$C \leqslant 8$	$G \leqslant 64$
$D \leqslant 28$	$H \leqslant 116$

Supply Constraints

The company has to meet certain orders. These minimum requirements per day are given in Table 2. Therefore, the company should make sure they meet these minimum requirements whereever possible. The minimum supply constraints are :

$A \geqslant 2$	$E \geqslant 6$
$B \geqslant 2$	$F \geqslant 14$
$C \geqslant 4$	$G \geqslant 28$
$D \geqslant 8$	$H \geqslant 14$

Objective Function

The objective is to maximize profits after satisfying all constraints. The profit per unit is the difference between the selling price and the cost of production given in Table 1.

The objective function is given by

$$\text{Profit } (Z) = 205A + 144B + 143C + 38D + 32.50E + 22F + 13.75G + 10.50H$$

The linear programming model for the above problem is

Maximize :

$$Z = 205A + 144B + 143C + 38D + 32.50E + 22F + 13.75G + 10.50H$$

Subject to:

R1:	A	$\geqslant 2$
R2:	B	$\geqslant 2$
R3 :	C	$\geqslant 4$
R4:	D	$\geqslant 8$
R5 :	E	$\geqslant 6$
R6 :	F	$\geqslant 14$
R7 :	G	$\geqslant 28$
R8:	H	$\geqslant 14$
R9 :	C	$\leqslant 8$
R10:	D	$\leqslant 28$
R11:	E	$\leqslant 36$
R12:	F	$\leqslant 42$
R13 :	G	$\leqslant 64$
R14:	H	$\leqslant 116$
R15 :	$125A$	$\leqslant 360$
R16:	$125B$	$\leqslant 360$
R17 :	$30C + 8.57D$	$\leqslant 360$
R18:	$30C + 6.67E$	$\leqslant 360$
R19 :	$30C + 5.63F$	$\leqslant 360$
R20:	$30C + 4G$	$\leqslant 360$
R21 :	$30C + 2.06H$	$\leqslant 360$
R22 :	$4700A + 3130B + 2610C + 1330D + 905E + 650F + 455G + 290H$	$\leqslant 126000$
R23:	$7A + 7B + 4.75C + D + 0.73E + 0.63F + 0.44G + 0.17H$	$\leqslant 150$

NOTE: R1 to R23 are the 23 constraints.

There are many computer programmes available for solving linear programming problems. One such programme is used to solve the above problem.

The solution to the above problem gives the daily production rate which maximizes profits. These depend on the available resources and other constraints.

However, what we are interested in at this stage is not the production rate but in the resource usage. The question asked is, what resources are fully utilized and what resources are in excess? This determined from the slacks in the various equations for the constraints.

It is observed that in the equation for the constraint for labour (R23), there is a slack equal to 7.14 manhours. This indicates that the provision of men is in excess of what is actually needed. The slack for the concrete and time are zero, which indicates that both these are fully utilized.

The required manhours per day, for the above production schedule is the difference between manhours provided and the slack, which is equal to

$$\frac{142.86}{8 \times 0.75} = 23.81$$
(say 24)

However, the company decides to provide 25 men.

VARIATIONS

The daily production schedule obtained above holds good if the manufacturing constraints remain constant. But time, labour and the supply of concrete will be subjected to many variations in actual practice.

Due to absenteeism there may not be one hundred per cent manhours available. The concrete supply may fall short due to breakdowns or shortages of raw materials. A total of eight hours of work may not be possible due to power failures, bad weather, strikes, meetings etc.

A change in any one of the constraints give a different set of values for the decision variables (daily production rate). Thus, the daily production rate will change when there is a change in any one or more of the resources.

The next step is to determine the daily production rate, maximizing profits for varying manufacturing constraints. This is done by forming linear programming models for various combinations of available time, labour and materials.

The manufacturing constraints are made critical now. In other words, the slacks in the equations for the constraints are removed. In the example, the slack in the equation for labour is 7.14 manhours. To remove the slack this amount is subtracted from the value in the right hand side of the equation. Thus, the new value in the right hand side of equation R23 becomes 150 – 7.14 = 142.86 manhours.

However, the slacks in the equations for the constraints in time and concrete supply (R15 to R22) are zero. The sales and the minimum requirement constraints (R1 to R14) are not altered. This is called the one hundred percent model or the critical model.

The resources in the one hundred percent model are:

Time = 360 minutes
Labour = 142.86 manhours
Concrete = 126,000 lbs.

Various combinations of different percentages of these resources are considered as shown in Tables 4 and 5 and the respective linear programming models analysed. One such model for 100% time, 90% labour and 80% concrete will be:

R15: $125A \leqslant 360$
R16: $125B \leqslant 360$
R17: $30C + 8.57D \leqslant 360$
R18: $30C + 6.67E \leqslant 360$
R19: $30C + 5.63F \leqslant 360$
R20: $30C + 4.00G \leqslant 360$
R21: $30C + 2.06H \leqslant 360$

R22: $4700A + 3130B + 2610C + 1330D + 905E + 455G + 290H \leqslant 100800$

R23: $7A + 7B + 4.75C + D + 0.73E + 0.63F + 0.44G + 0.17H \leqslant 128.5$

The rest of the constraints and the objective function remain the same.

NOTE: In computer programmes for the solution of linear programming problems, all right hand side values are punched in a seperate data card. This simplifies the computer work considerably as only one card has to be changed for every change of combination in time, labour and concrete.

Also combinations up to 30% at smaller intervals are recommended for practical purposes.

HOW TO USE TABLES 4 AND 5

Table 4 gives a schedule for 100% time and various combinations of labour and concrete. Table 5 gives a schedule for equal percentages of time and concrete combining with different percentages of labour. The reason for having these two types of tables will be clear when we consider the following cases.

Case A

Consider the case where only 20 men are available.

Labour = 20 x 8 x 0.75
= 120 manhours

Available resources as a percentage:

$$\text{Labour} \quad \frac{120}{142.86} \times 100 = 83.99 \quad (85 \text{ approx})$$

Time = 100
Concrete = 100

This indicates that 85% labour and 100% time and concrete are available for the day. Read either row 4 of Table 4 or row 4 of Table 5 for the day's schedule.

Case B

Consider the case when work starts only at 10.00 a.m. due to a power failure (loss of 2 hours) and 25 men are available.

Labour = 25 x 6 x 0.75
= 112.5 manhours

Time = 6 x 0.75 x 60
= 270 minutes

Concrete = (1800 x 7 x 6 x 0.75) + (1200 x 7 x 6 x 0.75)
= 94500 lbs.

Available resources as a percentage

$$\text{Labour} \quad \frac{112.5}{142.86} \times 100 = 78.78 \text{ (80 approx)}$$

$$\text{Time} \quad \frac{270}{360} \times 100 = 75$$

$$\text{Concrete} \quad \frac{94500}{126000} \times 100 = 75$$

This indicates that 75% of time, 80% of labour and 75% of concrete are available. It should be noted that the manufacture of concrete depends on the time available. Therefore any reduction in the percentage of time will cause an equal reduction in the concrete supply. Thus the percentages of time and concrete will be the same in the case of reduction of time. Read row 35 of Table 5 for the day's schedule.

Case C

Consider the case only 100,000 lbs of concrete can be supplied for the day due to a shortage of cement and 23 men are available.

Labour = 23 x 8 x 0.75
= 138 manhours

Concrete = 100,000 lbs.

Available resources as a percentage

$$\text{Labour} \quad \frac{138}{142.86} \times 100 = 96.59 \text{ (95 approx.)}$$

$$\text{Concrete} \quad \frac{100{,}000}{126{,}000} \times 100 = 79.36 \text{ (80 approx.)}$$

Time = 100

This indicates that the complete day is available and the only variations are labour and concrete. Read row 26 of Table 4 for the day's schedule.

Case D

Consider the case where work has to be stopped 1 hour 15 minuteas early due to a union meeting. Also there is a shortage of aggregates and only 110,000 lbs of concrete are available and 22 men have reported to work.

Labour = 22 x 6.75 x 0.75
= 111.37 manhours

Time = 6.75 x 0.75 x 60
= 303.75 minutes

Concrete : The amount of concrete available will be :

(1800 x 7 x 6.75 x 0.75) + (1200 x 7 x 6.75 x 0.75)
= 106312,5 lbs.

Note: Though 110,000 lbs of concrete are available only 106312.5 lbs. could be manufactured.

Available resources as a percentage

$$\text{Labour} \quad \frac{111.37}{142.86} \times 100 = 77.96 \text{ (75 approx.)}$$

$$\text{Time} \quad \frac{303.75}{360} \times 100 = 84.37 \text{ (85 approx.)}$$

$$\text{Concrete} \quad \frac{106312.5}{126000} \times 100 = 84.37 \text{ (85 approx.)}$$

This indicates that though there is a shortage of concrete due to the loss of time due to the meeting all the concrete available cannot be utilized. Thus, the time and concrete percentages are equal and row 24 of Table 5 gives the day's schedule.

Case E

Consider the case where work starts 45 minutes late due to bad weather and only 105,000 lbs of concrete and 24 men are available.

Labour = 24 x 7.25 x 0.75
= 130.5 manhours

Time = 7.25 x 0.75 x 60
= 326.25 minutes

Concrete : The amount of concrete available will be :

(1800 x 7 x 7.25 x 0.75) + (1200 x 7 x 7.25 x 0.75)

= 114,187 lbs.

However, only 105,000 lbs of concrete are available.

Available resources as a percentage.

$$\text{Labour} \quad \frac{130.5}{142.86} \times 100 = 91.34 \text{ (90 approx.)}$$

$$\text{Time} \quad \frac{326.25}{360} \times 100 = 90.62 \text{ (90 approx.)}$$

$$\text{Concrete} \quad \frac{105{,}000}{126{,}000} \times 100 = 83.30 \text{ (90 approx.)}$$

This is one case where both Tables 4 and 5 cannot be used. However, this type of case is not very common in practice.

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