INTERCITY ORIGIN DESTINATION ESTIMATION FROM LINK VOLUME COUNTS - A REVIEW

by

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Background

An Origin-Destination Matrix is one of the basic inputs in many travel analysis and planning studies. In urban transportation planning trip matrices are necessary in evaluating traffic management schemes, selecting most suitable planning options plus tactical and operational schemes. Defining bus routes, fixing bus line frequencies, setting one way streets, reorganising congested intersections are some examples of its use. At the national level too, origin-destination matrices are necessary to determine intercity travel demand to plan intercity roads, railways, planning intercity bus operations, fixing of bus and rail fares, location of airports etc.

Traditionally trip matrices are estimated by conducting a sample origin - destination survey. The total origin-distribution volumes are estimated for the future by calibrating a synthetic model - Evans (1970). If reasonable accuracy is required, a relatively large sample size should be considered.

Since the cost of surveys are high, in many developing countries the planners have found it difficult to evolve new transportation plans due to the non availability of origin destination matrices. Since the car ownership in developing countries is relatively low - [Jayawardena (1983)] a low priority is given to establish an auto travel intercity origin-destination matrix by actually conducting road side surveys. Also, in the recent past even in the developed countries the trend in transportation planning has shifted from strategic planning studies involving high budgets to tactical, and operational planning with limited budgets. These have led to the development of low cost techniques for estimating trip matrices from partial information. More recently, techniques and models for estimating trip matrices from traffic volume counts have been developed-Low (1972). Traffic counts are relatively inexpensive to collect, and are routinely collected for other purposes, such as maintenance of roads. Also, since there is a time difference between the conduct of survey actual implementation of the project, it has become very necessary to update the trip matrices even when actual surveys are conducted. This can be easily achieved by using the above mentioned techniques or models using current traffic volumes.

This paper reviews the current methods for intercity auto travel trip estimation using traffic volume counts and shows how they could be used to obtain an inter-city Origin Destination Matrix for Sri Lanka. The advantages and disadvantages of the two methods are later discussed.

Intercity Trip Estimation Methods from Traffic Volume Counts

There are two basic formulations in the estimation of intercity trip

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matrices from traffic volume counts. viz:

a) by calibrating demand models
b) the matrix estimation models using maximum entropy formalism.

Before the description of the above methods, the problem is first discussed in a general manner. Thereafter route choice methods are briefly described as this is an essential prelude to the application of the above methods.

A General Statement of the Problem

Let the intercity road transport network be defined by a set N (ϵ n) of nodes and a set A (ϵ a) of direct links (or arcs). In set N, some nodes are (i ϵ N) are zone origins and/or destinations, others are transhipment or connecting nodes.

Let the actual traffic demand between origin node i and destination node j be denoted by T_{ij}, and $[T_{ij}]$ is the trip matrix. When $[T_{ij}]$ are assigned onto all routes between i and j, they induce traffic flows V_a on some or all links in set A.

Let Ca(Va) be the cost function of link set A. This can be a generalized cost function or simply a delay function (travel time). Hence the cost of a route is the sum of the costs incured on all links forming r.

That is $U_r = \sum_{a} \epsilon A\, \delta_{ar}\, C_a(V_a)$

With $\delta_{ar} = 1$ if link a is in route r

and $\delta_{ar} = 0$ if link a is not in route r

Now the problem can be stated in the following manner.

If a reference trip matrix $[t_{ij}]$ and set of link traffic volumes are given, determine the best trip matrix $[T_{ij}]$ consistent with the observed arc flows. The reference trip matrix $[t_{ij}]$ may be (i) and outdated matrix giving information on the a priori cell probabilities from a traffic survey (ii) a matrix obtained from a simple gravity model using available data (iii) a matrix consisting of 1 or 0 cell values, or (iv) a combination of (i), (ii) or (iii).

Route Choice Analysis in Intercity Traffic

Basically there are two route choice selection (traffic assignment) methods

a) equilibrium assignment - (user equilibrium method)
b) proportional assignment

In the equilibrium assignment method, the generated traffic are assigned according to Wardrop's (1953) first principle. Thus trip makers are assumed to choose their routes in order to minimize their own individual travel costs. A good description of the user equilibrium methods using both deterministic and stochastic models are available in Kanafani (1983). However, this method is only suitable for congested road networks.

In proportional assignment, the proportion of traffic in each route is proportional to the route cost which does not depend on the traffic demand. The popular examples are, all or-nothing assignment in which the proportion of traffic in a route is either 1 or 0 for a particular origin-destination pair or multi-path assignment which distributes the demands onto routes according to a logit model. Dial (1971) used the following logit model for a multipath assignment model

$$n_{ijr} = \frac{e^{-\theta u_{ijr}}}{\sum_{k \epsilon R_{ij}} e^{-\theta u_{ijk}}} \qquad (2)$$

where = a calibrated parameter

n_{ijr} = proportion of traffic in route r for the o-d pair i-j

U_{ijr} = the cost of route r for the o-d pair i-j

R_{ij} = the routes taken by all i-j pairs in the system.

Proportional assignment is most suited for uncongested routes.

It can be assumed that intercity roads in Sri Lanka are uncongested and proportional assignment could be used in the estimation of auto travel between cities. Since route choice patterns are not available, only indirect calibration of a logit model may be done with the available traffic data. The intercity travel distances are available and travel times can be estimated from express bus travel schedules. All or nothing assignment can also be adapted without any difficulty.

Trip Matrix Estimation by Calibrating Linear and Non-Linear Demand Models

The linear demand models were used by Low (1972), Overgard (1972), Symon et.al. (1976) and Holm (1975) is of the form,

$$T_{ij} = \beta_0 + \Sigma \beta_m X_{ijm} \qquad (3)$$

with the traffic x_{ijm} from i to j for purpose in taking a gravity type product form

$$X_{ijm} = f_m(A_i m, B_{jm}, C_{ij}) \qquad (4)$$

where A, B are exogenius variables such as population, employment [Low (1972)], the percentage of single family households [Overgaard (1972)] a recreation attractiveness index (Symon et.al.(1976)].

Using traffic volumes, parameter values were determined and T_{ij} values were then calculated.

In general, linear models have been found to produceless realistic results than non-linear models [Lamarre (1977)]. Non-linear models have been used by Wills (1971), Robillard (1974), Hogberg (1976). Symon et.al. (1976) used a linear demand model for the initial calibration and then calibrated a non-linear model to estimate the intercity motor travel in Australia. However, the calibration process of the non-linear demand models have not been done in a rigorous manner.

Hogberg (1976) used the following non-linear demand model taken from Bexelius et.al.(1969) to estimate travel between cities in a hypothetical network.

$$T_{ij} = A_{i1} b_1 \frac{A_{j2} f(d_{ij})}{\sum A_{k2} f(d_{ik})} + A_{i1} b_2 \frac{A_{j1} f(d_{ij})}{\sum A_{k1} f(d_{ij})} + A_{i2} b_3 \frac{A_{j2} f(d_{ij})}{\sum A_{k2} f(d_i k)} \qquad (5)$$

where $f(d_{ij}) = d_{ij}^{b4} \exp [b_s \ln(d_{ij})^2]$

The three components in the systematic part of the model stand for three sub-classes of trips: home-work, home-home and work-work.

A_i and A_j are the number of inhabitants and number of employees and d_{ij} the distance between i and j.

When all or nothing traffic assignment model is used,

We have = 1 if the least cost route from i to j passes through a
= 0 otherwise

Let [N] be the matrix containing all the above data

Hence matrix [N] has elements <u>n²</u> <u>x</u> <u>a</u>

Let [G] be a <u>axp</u> matrix which is

used to indicate the p links on which traffic is counted on first p links,

then $[G] = [I/0]$ where I is a pxp identity matrix. and 0 is an px(a-p) zero matrix

Hence, Computed traffic volumes

$$[V^*] = [G]^T [H]^T [T_{ij}] \quad (6)$$

which is a pxa matrix

If the actual traffic volumes = [V]; (pxa matrix)

then the least square objective function is:

$$Q = \left[[G]^T[H]^T[T]-[V]\right]^T \left[[G]^T[H]^T[T]-[V]\right] \quad (7)$$

Hogberg (1976) used an algorithm for non-linear regression suggested by Marquadt (1963) to calibrate the model given inl. 5 to minimize the objective function 7. In Marquadt (1963) algorithm a maximum neighbourhood method is developed, which in effect, performs an optimum interpolation between the Taylor Series method and the gradient method as discussed in Malinvaud (1980). The interpolation is based upon the maximum neighbourhood in which the truncated Taylor Series gives an adequate representation of the non-linear model. Wills (1971) suggested a product form demand model

$$T_{ij} = \beta_0 \prod_m (A_{im} B_{jm})^{\beta_m} \quad (8)$$

Where A and B are the n^{th} explanatory variables associated with i and j. He used this model to estimate the inter-city travel for the provinces of Quebec and British Columbia in Canada.

Trip Estimation by Maximum Entropy Models

Jaynes' (1957) proposed that Shanon's (1948) measure of uncertainty can be used to define values for probability distributions in problems of statistical inference. In particular, if the given information is in the form

$$\sum_i p_i = 1 \quad \text{where } p_i = \text{the probability of occurance } i \quad (9)$$

$$\sum p_i f_a(x_i) = a_j \quad j = 1,2 \quad (9a)$$

the "maximally non-commital" probability distribution is the set of p which maximizes

$$E = -K \sum p_i \log p_i \quad (10)$$

where K is a scaling factor. Any other choice would amount to arbitrary assumptions of information.

If p_{ija} is the probability that an observed unit of flow has been produced by a trip associated with the i^{th} origin and j^{th} destination pair on arc a,

$$\text{then } p_{ija} = \frac{Z_{ija}}{\sum_{a \in L} \bar{V}_a} \quad (11)$$

where

Z_{ija} = the flow on arc a, produced from traffic demand $T_{ij,ij}$ if $[t_{ij}]_w$ is the available a priori information, then q_{ija}, the corresponding a priori probability is given by

$$q_{ija} = \frac{t_{ij} n_{ija}}{\sum_{ija\, s} t_{ij} n_{ija}} \quad (12)$$

Using Jaynes (1957) maximum entropy formalism, the least biased probability distribution $[p_{ija}]$ is that distribution which maximize the entropy function subject to the given information

That is, Maximize

$$-\sum_{ija\ s} p_{ija} \log \frac{(p_{ija})}{q_{ija}} \quad (13)$$

$$\text{subject to} \sum_{ia \in s} p_{ija} = 1 \quad (13a)$$

$$\sum_{ija \in S} P_{ija} \; f_a(Z_{ie}) = \frac{\bar{V}_a}{\sum_{a \in L} \bar{V}_a} \qquad (13b)$$

$$\text{where } f_a(Z_{ie}) = 1 \text{ if } a = e$$
$$= 0 \text{ otherwise} \qquad (13c)$$

Substituting $T_{ij}\,n_{ja}$ for Z_{ija}, eliminating constant term from the objective functioned deleting the redundant equation

$$\sum_{ija \in S} P_{ija} = 1$$

the following weighted entropy problem in variables T_{ij}, $ij \in w$ is obtained.

$$\text{Maximize} \quad \sum_{ij \in w} K_{ij} T_{ij} \log \frac{T_{ij}}{t_{ij}} \qquad (14)$$

$$\text{subject to} \quad \sum_{i \in w} T_{ij}\, n_{ija} = V_a \quad a \in L \qquad (14a)$$

$$T_{ij} > 0, \; T_{ij} = 0, \; ij \in w \qquad (14b)$$

$$\text{where} \quad K_{ij} = \sum_{a \in L} n_{ija} \qquad (14c)$$

The solution to the above set can be expressed in the form of a multi-proportional model.

$$T_{ij} = t_{ij} \prod_{a \in L} \alpha^{n_{ija}} / K_{ij}, \; ij \in w$$

$$\text{where } \alpha^{n_{ija}} = e^{-\frac{1}{n_{ija}}} \times e^{-\lambda a}$$

λ_a are the Lagrangian multipliers

The original idea of deriving a trip matrix from the least biased estimates of the numbers of arc trips was put forward by Van Zuylen and Willumsen (1980). Willumsen (1978) solved the multiproportional problem following Marchland (1977) using an all or nothing assignment.

For non-congested network, all or nothing assignment is accepted as a reasonable allocation of traffic demand in the network

For congested networks, where multiple routes are utilized between any given origin-destination pair, there is no known method for accurately estimating the traffic proportions separately from the traffic demand. This led to the development of models to estimate the traffic proportions by integrating trip demand estimation and network equilibrium traffic assignment into one process.

Summary

This paper has reviewed models and methods most suitable for inter-city travel estimation for Sri Lanka from observed link volume counts. The two basic modelling approaches that have been suggested are:

a) the calibration of demand models
b) the matrix estimation models

Calibration of a demand model helps to estimate the future traffic with a knowledge of the exogenious variables. It is unable to incorporate all the available information. However, a full matrix can be obtained with limited data.

Maximum entropy model has the advantage of using all the available information and obtaining the most suitable origin-destination matrix. However, sufficient information should be available to produce the full matrix. Otherwise only a partial matrix is produced.

Nguyen (1984) has found that applications of the current models are still limited as traffic counts cannot capture all the structural particularities of the trip matrix so that the accuracy of the resulting estimates is highly sensitive to the input trip matrix. Hence any planning work undertaken with these estimations should be done with utmost care.

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