PREDICTION OF THE PERFORMANCE OF SINGLE-PHASE INDUCTION MOTORS BY DIGITAL COMPUTER

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Synopsis

This article describes the development of a flexible computer program based on the application of the theory of Symmetrical Components to the general unbalanced two-phase induction machine. This program can be used to predict the performance and also for selecting the best combination of design parameters for a wide variety of induction motor types. The computer predictions are checked by actual tests on a single-phase induction motor and the results of a typical motor calculations are presented and discussed.

1. Introduction

The behaviour of single-phase and two-phase induction machines are usually explained by either the Cross-field theory[1,2] or the revolving field theory.[3] Although the final results are the same in both cases a completely different approach is used in each of these methods. Based on these methods calculation procedures[4,5] have been presented for predicting the motor performance. These methods are quite cumbersome and time-consuming and involve the use of several empirical relationships. Ismail et al[6] have presented an analogue model of the motor, one distinct advantage of which is that changes in degree of magnetic saturation can be included.

An elegant analytic procedure using the concept of symmetrical components has been developed by Suhr[7] from which a simple but accurate equivalent circuit can be deduced for the general two-phase induction motor. This method, while providing a clear intuitive understanding of the phenomena involved, forms an excellent base for numerical computations. All the quantities needed for the procedure can be obtained either from design data or from experiments.

2. Theory[7]

Suffixes and Symbols.

1	main (reference) winding
2	auxiliary (start) winding quantities referred to main winding.
r	rotor
+, —	positive and negative sequence components of quantities in main (reference) winding
x_m	magnetising reactance
$z_1 = r_1 + jx_1$	leakage impedance of main winding
$z_a = r_a + jx_a$	leakage impedance of auxiliary winding
$z_c = r_c - jx_c$	impedance of the capacitor (equivalent series representation)
$r_r + jx_r$	leakage impedance of rotor at standstill referred to (stator) main winding
s	slip (p.u.)
V_c	voltage across the capacitor
a	effective turns ratio N_a/N_1
w_0	synchronous speed

Consider a general two-phase induction machine in which the axes of the stator windings are in quadrature, and sinusoidal voltages V_1 and V_2 of the same frequency applied to the stator windings. Expressing the voltages and currents in terms of the symmetrical components[7]

$$V_+ = 1/2(V_1 - jV_2/a) \quad (1)$$

$$V_- = 1/2(V_1 + jV_2/a) \quad (2)$$

$$I_+ = 1/2(I_1 - jaI_2) \quad (3)$$

$$I_- = 1/2(I_1 + jaI_2) \quad (4)$$

The corresponding inverse relationships are:

$$V_1 = (V_+ + V_-) \quad (5)$$

$$V_2 = ja(V_+ - V_-) \quad (6)$$

$$I_1 = (I_+ + I_-) \quad (7)$$

$$I_2 = j/a(I_+ - I_-) \quad (8)$$

For the main winding,

$$V_1 = I_1 Z_1 + (E_+ + E_-) \quad (9)$$

where E_+ and E_- are the induced e.m.fs in the main winding due to the forward flux wave and backward flux wave respectively. These e.m.fs can be expressed in terms of the respective impedances as

$$E_+ = I_+ . z_+, \text{ and } E_- = I_- . z_-,$$

where z_+ and z_- are the input impedances shown in Fig. 1b and 1c.

Similar relationships exist for the auxiliary winding. On substitution of equations 5 to 8 in these and after algebraic simplification,

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$$I_+ = \frac{(z_s + z_-)\,V_+ + (z_d)\,V_-}{Det} \qquad (10)$$

$$I_- = \frac{(a_d)\,V_+ + (z_s + z_+)V_-}{Det} \qquad (11)$$

where $z_s = 1/2(z_2 + z_1)$, $z_d = 1/2(z_2 - z_1)$,

and $Det = (z_s + z_+)(z_s + z_-) - z_d^2$

The gross steady state electromagnetic torque T_g is the difference between the torques due to the forward and backward fields, and is expressed as

$$T_g = 2/\omega_0 (I_+^2 r_+ - I_-^2 r_-) \qquad (12).$$

It is possible to eliminate the torque due to the backward field at any given value of slip, by suitable choice of the auxiliary winding parameters. Expressing Z_2 as,

$$Z_2 = \left[(r_a + jx_a) + (r_c - jx_c)\right] 1/a^2$$

and equating I_- to zero, (Equation 11), we get

$$r_c = a(x_1 + x_+) - (r_a + a^2 r_+) \qquad (13)$$

$$x_c = a(r_1 + r_+) + (x_a + a^2 x_+) \qquad (14)$$

Note that r_+, and x_+ are functions of the slip.

3. The Computer Program

A program was developed using the above theory for the general unbalanced two-phase induction motor having winding axes in quadrature. The program is designed to give as output, the currents (in magnitude and phase) in each of the stator windings, the components of torque due to forward and backward fields, the voltage across the capacitor etc., at any value of slip. The performance characteristics of most of the commonly used varieties of induction motors such as the single-phase induction motor of the split phase type, capacitor start induction run, split capacitor, two value capacitor, the two-phase a.c. control motor, and the influence of variation of design parameters on these characteristics can be investigated by suitable control statements incorporated in the general program. Some of these special cases are listed below.

(i) Single-phase operation :
In this case $V_1 = V_2$; hence $V_- = \text{conj}(V_+)$

(ii) Starting conditions of a single-phase motor :
Here $S = 1$; hence $z_+ = z_-$
Let this be represented by $Z_0 = R_0 + jX_0$
Then $I_1 = V/(Z_0 + z_1)$, $I_2 = V/a^2(Z_0 + z_2)$, and starting torque $= 2a\,\omega_0 \cdot I_1 I_2 R_0 \sin\alpha$, where α is the angle by which I_2 leads I_1.
When the capacitance is varied the locus of I_2 is a circle and the starting torque is proportional to the normal component of I_2 with reference to I_1 (see Fig. 2b).

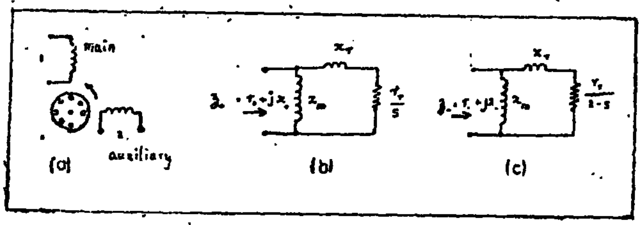


Fig. 1. (a) 2 phase induction motor
(b) Impedance to the forward field
(c) Impedance to the backward field

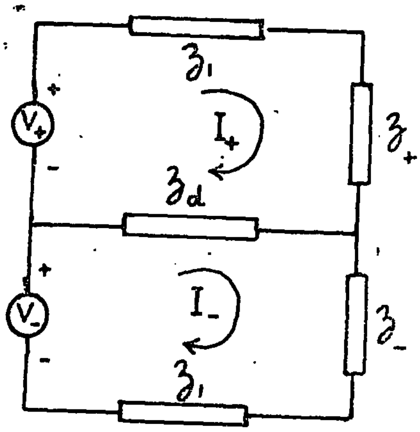


Fig. 2a

Fig 2a. Equivalent circuit of the 2 phase induction motor

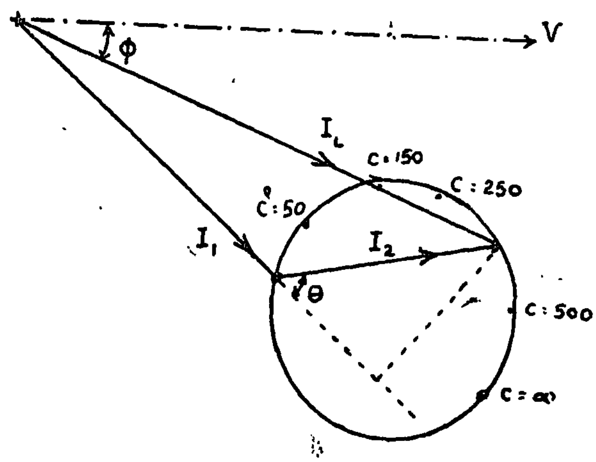


Fig. 2b

Fig 2b. Locus of motor line current Ia as the capacitance is varied

(iii) Split phase motor :
For this case substitute $r_c = x_c = 0$.

(iv) Capacitor start, induction run type :
For the starting condition, the appropriate values of r_c and x_c are inserted. For the running condition direct substitution of $z2 = \alpha$

yields an indeterminate form for equations (10) and (11). Rearrangement of these equations give

$$I_{+} = I_{-} = \frac{V}{(r_{+} + r_{-} + 2r_{1} + j(x_{+} + x_{-} + 2x_{1}))}$$

(v) Two value capacitor motor :
The appropriate values of r_c and xc are used for the speed ranges before and after the operation of the switch.

Table 1

$r_1 + jx_1$	$= (0.96 + j1.09)\Omega$	$r_r + jx_r$	$= (1.14 + j1.10)\Omega$
x_m	= 28.0 ohms	f	= 50 Hz
Core loss	= 210 watts	No load friction and windage	= 30 watts
Starting Capacitor	= 270 uF	a	= 1.05
Rated Current	= 19 A	Rated speed	= 1430 r.p.m.

Parameters of the 3-h.p, 230-V, 4-pole Westinghouse Inductor Motor (obtained from tests)

Table 2

$r_1 + jx_1$	$= (2.02 + j2.79)\Omega$	$r_a + jx_a$	$= (7.14 + j3.22)\Omega$
$r_r + jx_r$	$= (4.12 + j2.12)\Omega$	a	= 1.18
$r_c + jx_c$	= 3.00 − j14.5 (starting)	x_m	= 66.8
$r_c + jx_c$	= 9.00 − j172 (running)	f	= 60 Hz
Core loss	= 24 watts	no load friction loss	= 13 watts

The constants of a 1/4 h.p., 110-V, 4-pole induction motor. (ref.3,8)

4. Results and Discussion

The parameters of a 3 h.p., single-phase capacitor start induction motor were determined experimentally and these were used as data for the Computer Program to predict the motor performance. The predictions were checked by comparing these with the results of actual load test on the motor (Table 1 and Fig. 3). The validity of the program was thus established.

One interesting outcome of these tests concerns the iron losses of the motor. There is some difference of opinion among authors [2,5,7,8] as to what fraction of the total iron losses of single-phase motors should be considered as supplied by the rotor. Some authors [7,8] subtract the entire iron loss from the gross electromechanical output, while others[5] consider the entire loss as being supplied by the stator. In reality, on account of the backward rotating m.m.f. wave, there are double frequency pulsations in the rotor, and although its amplitude is much less than that of the forward wave, do cause an appreciable iron loss in the rotor, which reduces the rotor output power. An investigation revealed that for the 3 h.p. motor tested, approximately

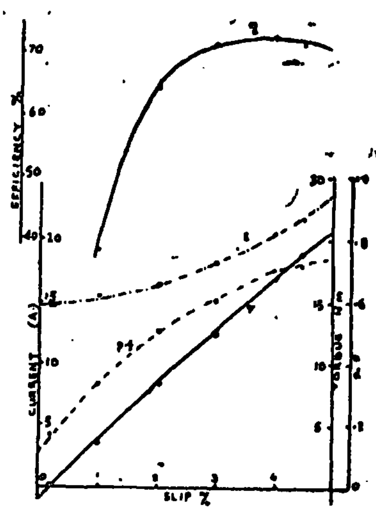


Fig. 3

Fig. 3. Characteristics of the 3-h.p, 1-phase (Westinghouse) motor (Test results are shown by dots)

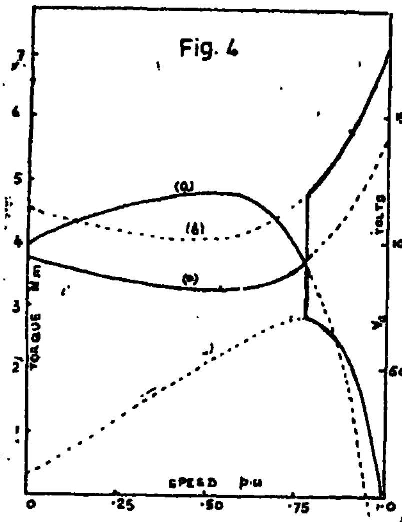


Fig. 4. Starting and running characteristics of the 1/4 h.p. motor (a) C — 184 uF (b) C — 15.5 uF

30% of the total iron loss of 210 watts does indeed come from the rotor output.

Next, the constants of the 1/4 h.p., two value capacitor motor listed in references 3 and 8, were used to predict the starting and running characteristics (Table 2 and Fig. 4). The influence of varying the value of the starting capacitor and the value of the running capacitor on these characteristics were then studied as typical examples of the use of the program developed. (Figs. 5, 6 and 7).

The value of the starting capacitance was varied through a wide range and its effects on the starting torque, torque/amp, the voltage across it, are shown in Fig. 5. The loci of the currents are shown in Fig. 2b. Theoretically if r_c had been kept constant, the locus is a circle. However in the computation, r_c was considered to vary linearly with x_c (i.e. the loss angle of the capacitor was kept constant at the design value). It will be seen that

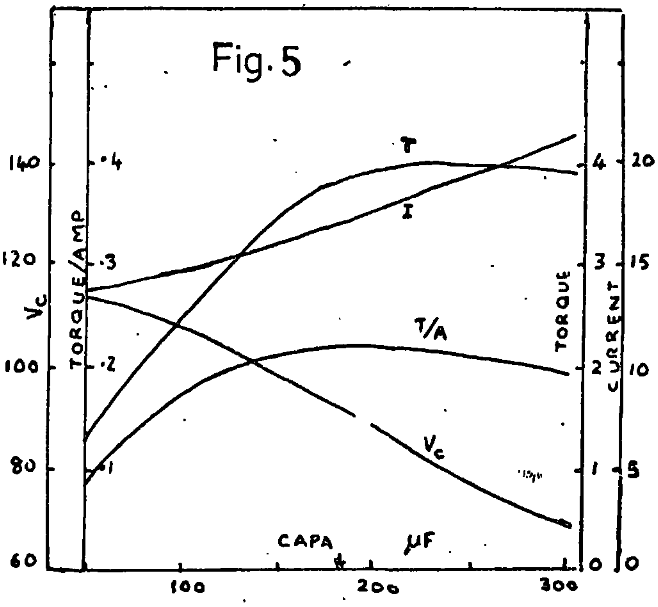


Fig 5. Starting Characteristics of the 1/4 h.p. motor

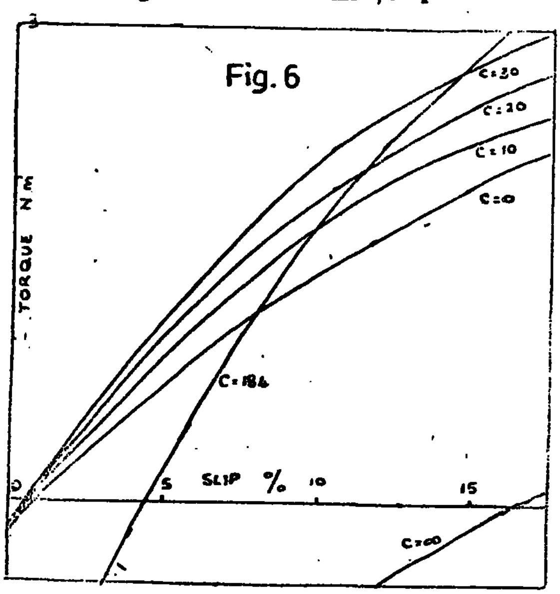
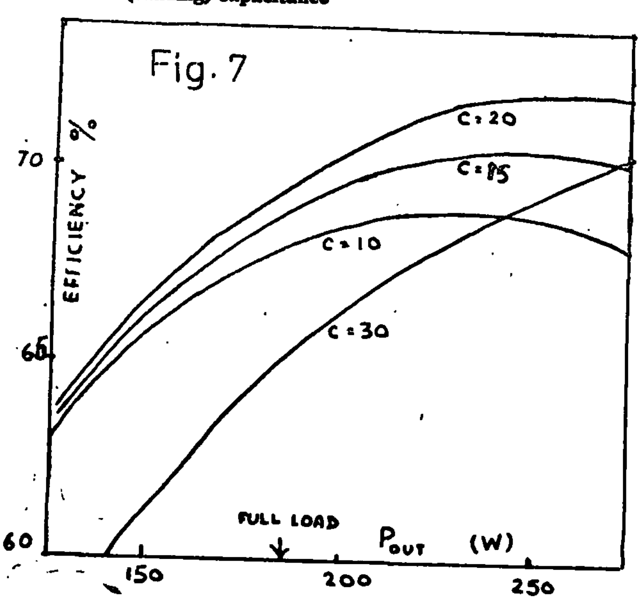


Fig 6. Variation of motor torque with slip for different values of (running) capacitance



the maximum starting torque occurs when C = 310 μF but there is practically no increase in Torque/Amp for values of C greater than 200 μF. This compares favourably with the design value of 184 μF.

The influence of the parameters of the auxiliary winding on the running characteristics can be seen from Figs. 6 and 7. The capacitive reactance was varied from ∞ to zero. The curve C = O, represents the characteristic of the motor running with only the main winding. The curve C = ∞ shows the effect of short circuiting the running capacitor (i.e. xc = O). From the latter curve, we can confirm the result that the auxiliary winding by itself (with the running capacitor shorted) does produce a large negative torque in the region of the full load speed and that the value of the running capacitor should be chosen rather carefully. Too small a value of capacitor causes a reduction of torque and efficiency. Too large a value also causes large slips and loss of efficiency. The optimum value was found to be approximately 22 μF. (c.f. the design value of 15.4 μF) leading to a full load efficiency of 70%.

The extra resistance and the capacitive reactance which when introduced into the auxiliary winding would eliminate the torque due the backward field, were computed for the 1/4 h.p. motor and are given below.

Slip p.u. ..	—	0.02	0.04	0.06
Resistance Ω	—	23.50	9.68	−8.44
Capacitive Reactance Ω		106.0	99.1	91.6

In practice, in the motor design procedure no conscious attempt is made to eliminate this negative torque because the required value of extra resistance often turns out to be negative or the extra component values are such that, it does in fact lead to a decrease in overall efficiency owing to the excessive copper losses. Besides in a well-designed motor this negative torque component seldom exceeds 10% of the positive component in the normal operating range of slip.

5. Conclusions

The development and application of a flexible computer program for the accurate determination of the characteristics of a wide variety of Induction Motors has been presented and discussed. It is expected that this program would be of immense value to the designer whose task is mainly one of striking a judicious compromise of numerous conflicting requirements involved such as (i) high starting torque, (ii) high starting torque/ampere, (iii) low capacitor voltage, (iv) high efficiency and power factor and the most important factor, the overall cost. On account of the complex interaction among the two stator windings and the rotor, the machine performance can be predicted accurately only when a complete mathematical model has been set up. Such a model could easily be set up as a program in digital computer. This program would provide quick method of checking the preliminary design and also illustrating the influence of the variation of one or more parameters.

6. Acknowledgements

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7. References

1. Pushtein, A. F. and Lloyd, T. C., "*Crossfield Theory of the Capacitor Motor*", Trans. AIEE, Vol. 60 (1941), pp. 58–61.

2. Ozawa, A., "*The Analysis of Single-Phase Induction Motor by Crossfield Theory*", Elect. Eng. in Japan, Vol. 90 (1970), No. 3, pp. 140–151.

3. Morril, W. J., "*The Revolving Field Theory of the Capacitor Motor*", Trans. AIEE (1929), pp. 614–629.

4. Veniott, G. G., "*Performance Calculation of Induction Motors*', Trans. AIEE, Vol. 51 (1932), pp. 743–755.

5. Trickey, P. H., "*Performance Calculation on Capacitor Motors*", Trans, AIEE, Vol. 60 (1941), pp. 73–76.

6. Ismail, M. K. E. and Adkins, B., "*Performance Calculation of the Capacitor Motor Using the Transformer Analogue Analyser*", Proc. IEE, Vol. 106 (1959), pp. 175–182.

7. Suhr, F. W., "*Symmetrical Components as Applied to Single-Phase Induction Motors*", Trans. AIEE (1945), Vol. 64, pp. 651–656.

8. Lyon, W. V. and Kingsley, C., "*Analysis of Unsymmetrical Machines*", Trans. AIEE, Vol. 55 (1936) pp. 471–476.

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අසංතුලිත ද්විකලා ප්‍රේරණ මෝටර් වල ක්‍රියාකාරිත්වය හැදෑරීම, සමමිතික සංරචක භාවිතයෙන් කලහැකිය. මෙම ලිපියෙන් විස්තර වනුයේ එම මූලධර්මය මත පදනම්වූ නම්‍ය ආගණක ප්‍රක්‍රමයකි. ප්‍රේරණ යන්ත්‍ර වර්ග ගණනාවකම ක්‍රියාකාරිත්වය පෙරැයීමටත්, වඩාත්ම සුදුසු නිර්මාණ අංග තෝරා ගැනීමටත් මෙම ප්‍රක්‍රමය උපයෝගී කරගත හැකිය. ආගණකයකින් ලබාගත් ප්‍රතිඵල, තනිකලා මෝටරයක් සඳහා කරනු ලබන භෞතික පරීක්ෂයකින්, නිරවද්‍යබව අවේක්ෂණය කර ඇත. ප්‍රරූප මෝටරයක් සඳහා භාවිතාවන ගණනය කිරීමක්ද මෙහිදී සාකච්ඡා කර ඇත.

தூண்டல் மோட்டார் தொழிற்திறன் உய்த்தெறிதல்

ஜே. பீ. எக்ஸ். டிவோட்டா

இக்கட்டுரை, சமநிலையற்ற இரு நிலைமை தூண்டல் மோட்டாரை சமசீர் பகுதிக் கொள்கையை பிரயோகித்து, மிகவும் வளைவுள்ள இலக்கக்கணி நிகழ்ச்சித் திட்டமொன்றின் விரிவாக்கலை விளக்குகின்றது. பலவகைப்பட்ட தூண்டல் மோட்டர்களின் தொழிற்திறனை உய்த்தெறிவதற்கும், திட்டமிடல் சாராமாறிகளின் சிறந்த அளவுகளை தெரிவு செய்வதற்கும், இந்த நிகழ்ச்சித் திட்டத்தை பயன் படுத்தலாம். ஒருநிலை மோட்டரின் பரிசோதனை முடிவுகளை இலக்கக்கணி திட்டத்தின் எதிர்பார்ப்புகளுடன் செய்வை பார்க்க படுவதையும், மோட்டரின் பணிப்பின் பெறுகளையும் இதில் கலந்துரையாடபடுகிறது.

Errata

The following corrections relate to the article on 'Resistance-type Vertical-Axis Wind Turbines', published in page 30 of the March 1978 issue of 'Engineer'. The editor regrets these errors.

The title should read 'Resistance-type Vertical-Axis Wind Turbines: The Influence of Reynolds Number and Aspect Ratio on Their Performance.'

Rec, R p, Re o and R tip should read Re_c, R_p, Re_o and R_{tip}, respectively, throughout.

Line 12, col. 1, p. 31: 'ratio' should read 'ratios'.
Line 6, col 1, p. 32: 'those' should read 'these'.
Figure 5, notation: p ese: t w ɔrk ⚯
Snankar (2-bladed rotor) —•—
Shankar (3-bladed rotor) — —