EFFECTS OF OPPORTUNE MAINTENANCE IN A SINGLE MACHINE JOB SHOP

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Abstract

In this paper, the notion of opportune maintenance is presented and its effects in a single machine job shop are discussed. The job shop without opportune maintenance is modelled as a single serve queue and the job shop with opportune maintenance as a single server queue with single 'server vacations'. The results available for these models are used to derive the mean inventory in the job shop and the cost rates with and without opportune maintenance. A numerical example is given to illustrate the effects of opportune maintenance.

Introduction

In a recent paper in the Quarterly Journal of the Institution of Engineers, Sri Lanka, Hall.[2] pointed out the importance of Terotechnology in production and related systems which consist of components that can fail. In terms of maintenance policies, he presented three basic options to protect the investment in systems that are subjected to failure. They are:

1. Wait and see—or emergency breakdown maintenance,
2. Preventive policy—using planned maintenance,
3. Life prolonging policy—implies the use of condition monitoring techniques and also the application of those materials compatible to the operating conditions.

One important policy that could be added to this list is opportune maintenance policy. In a complex production system, if one component fails some other components of this system may be forced down due to insufficient storage space (called blocking, refer to Buzacott and Shanthikumar.[1] or forced idling because there are no jobs available to be processed. In such cases while the failed component is being repaired (emergency breakdown maintenance) we can carry out preventive maintenance (called opportune maintenance) on those components that are blocked or idling. This opportune maintenance is in effect an unscheduled preventive maintenance policy. Therefore opportune maintenance can also be listed as part of preventive policy.

In this paper we will study the effects of opportune maintenance in a single machine job shop. In a single machine job shop, opportune maintenance may be carried out when the machine is idling. Next we describe the model and derive the cost rates for the job shop with and without opportune maintenance.

The Model

In this section we consider a job shop with a single machine. Job orders arrive to this job shop according to a Poisson process with an average rate of λ orders per day. Each job requires a processing time B at the machine. B is assumed to be a random variable (rv) with cumulative distribution function (c.d.f.) B(·), mean E(B), and second moment $E(B^2)$. We will also assume that the life time of the machine is exponentially distributed with mean $1/a(\delta)$ days without opportune maintenance and with mean $1/\alpha'(\theta)$ days when the average number of opportune maintenance per day is θ. Note that $\alpha = \alpha(o)$. The repair time of the machine is assumed to be a r.v R with c.d.f. R(·) and the opportune maintenance time is a r.v T with c.d.f. T (.).

C_I is the inventory cost per job per day.

C_R is the cost of one repair.

C_M is the cost of one opportune maintenance.

Even though each job requires a service time B, due to the machine failures, the actual time S required to complete the service of the job will be more than B. By counting the number of failures during the service of a job, it can be shown that the c.d.f. S(.) of the actual service time S needed to complete service of a job is given by

$$\tilde{S}(s) = \tilde{B}[s + \gamma - \gamma\tilde{R}(s)] \qquad (1)$$

where $\tilde{S}(.)$, $\tilde{B}(.)$, and $\tilde{R}(.)$ are the Laplace Stieltjes transforms (LST) of S(.), B(.), and R(.), respectively, and $1/\gamma$ is the mean of the exponential life time of the machine. With opportune maintenance, $\gamma = \alpha'(\theta)$ and without opportune maintenance $\gamma = a$. Now

$$E(S) = -\lim_{s\to 0}\frac{d}{ds}\tilde{S}(s) = [1 + \gamma E(R)]E(B) \qquad (2)$$

and

$$E(S^2) = \lim_{s\to 0}\frac{d^2}{ds^2}\tilde{S}(s) = \gamma E(B)E(R^2) + [1+\gamma E(R)]^2 E(B^2) \qquad (3)$$

Case 1. No opportune maintenance.

When there is no opportune maintenance the single machine job shop can be modelled as a single server queue M/G/1 with Poisson arrival process with rate λ and service time S with $\gamma = a$ and general c.d.f. Then from the results available for M/G/1 in Kleinrock[3], page 187, equation 5.67] we have

$$E(N_\infty) = \lambda E(S_\infty) + \frac{\lambda^2 E(S_\infty^2)}{2[1 - \lambda E(S_\infty)]} \qquad (4)$$

where $E(N_\infty)$ is the mean inventory (number of jobs) in the job shop and $E(S_\infty)$ and $E(S_\infty^2)$ are obtained from (2) and (3) with $v = a$. That is

$$E(S_\infty) = [1 + \alpha E(R)]E(B)$$

and

$$E(S_\infty^2) = \alpha E(B)E(R^2) + [1 + \alpha E(R)]^2 E(B^2)$$

Since the mean life time of machine is $1/a$, the mean number of failures during the service of a job in $aE(B)$. Since the arrival rate of the jobs is λ the average number of repairs per day is $\lambda aE(B)$. Then the total cost of operation per day is TC_∞ given by

$$TC_\infty = E[N]_\infty C_I + \lambda\alpha E(B)C_R \tag{5}$$

Case 2. Opportune maintenance at every time the machine begins idling.

The single machine job shop with opportune maintenance at every time the machine begins idling can be modelled as a single server queue M/G/1 with single "server vacations" (refer to Levy and Yachiali[4], and Shanthikumar[8] and with arrival rate λ, vacation length T and service time S with $v = a(\epsilon)$. If the machine begins idling on the average every E(C) days, we have $\epsilon = 1/E(C)$. This average E(C) is the expected length of the underlying regeneration cycle of this M/G/1 queue and Shanthikumar[8] has derived this on page 21 of ref.[8]. It is

$$E(C) = \frac{\lambda E(T) + \tilde{T}(\lambda)}{\lambda[1 - \lambda E(S_1)]}$$

where

$$E(S_1) = [1 + \alpha(\epsilon)E(R)]E(B) \tag{6}$$

Therefore,

$$\epsilon = \frac{\lambda[1 - \lambda E(S_1)]}{\lambda E(T) + \tilde{T}(\lambda)} \tag{7}$$

The mean inventory $E(N_1)$ can be obtained from (21) of ref.[4]. It is

$$E(N_1) = \lambda E(S_1) + \frac{\lambda^2 E(S_1^2)}{2[1 - \lambda E(S_1)]} + \frac{\lambda^2 E(T^2)}{2[\tilde{T}(\lambda) + \lambda E(T)]}$$

where $E(S_1^2) = a(\epsilon)E(B)E(R^2) + (1 + a(\epsilon)E(R))^2 E(B^2)$

Similar to Case 1 it can be shown that the average number of repairs per day is $\lambda a(\epsilon)E(B)$. Then the total cost of operation TC_1 per day with opportune maintenance is given by

$$TC_1 = E[N_1]C_I + \lambda\alpha(\epsilon)E(B)C_R + \epsilon C_M \tag{8}$$

Comparison of Cost Rates

If a job shop had been operated up to now with no opportune maintenance, we will have the necessary data to estimate λ, a, E(B), $E(E^2)$, E(R) $E(R^2)$, C_I, C_R, and TC_∞. Now in order to decide whether opportune maintenance will bring us any benefit we will have to estimate TC_1. To do this we need ϵ, $a(\epsilon)$, T(.), and C_M. While T(.) and C_M can be estimated fairly comfortably, the estimation of ϵ and $a(\epsilon)$ will lead to problems due to their inter-relationship. From equations (6) and (7) we can see that the mean life of the machine $1/a(\epsilon)$ will depend on the average number of opportune maintenance ϵ per day and ϵ will depend on $a(\epsilon)$. One possible way to evaluate ϵ and $a(\epsilon)$ is to get a functional relationship between $a(\epsilon)$ and ϵ and use (6) and (7) to evaluate them. Once these are evaluated we can compare TC_1 and TC_∞ and decide whether or not to use opportune maintenance. The difference $TC_\infty - TC_1$ is given by

$$TC_\infty - TC_1 = [E(N_\infty) - E(N_1)]C_I + \lambda E(B)[\alpha - \alpha(\epsilon)]C_R - \epsilon C_M \tag{9}$$

Next we will consider a special case of the relationship between ϵ and $a(\epsilon)$ and present a numerical example.

A Ratio-k Reduction Model

In a ratio-k reduction model we assume that for each opportune maintenance performed on the machine the number of failures will be reduced by k. Let N(t) be the number of failures observed in a time period of t days without opportune maintenance. Then if we give $N_0(t)$ number of opportune maintenance over the next t days, on the average we would expect $N(t) = \max(N(t) - kN_0(t), 0)$ failures with opportune maintenance. Now dividing the above equation by t and taking the limit as $t \to \infty$ we get

$$\alpha(\epsilon)\lambda E(B) = [\alpha\lambda E(B) - k\epsilon]^+ \tag{10}$$

where $(x)^+ = \max(x,0)$. Now using (6), (7), and (10) we can solve for ϵ and $a(\epsilon)$.

A One-to-One Reduction Model

A one-to-one reduction model is a special case of the ratio-k model where $k = 1$. Then

$$\alpha(\varepsilon)\,\lambda E(B) = [\alpha\lambda E(B) - \varepsilon]^{+} \qquad (11)$$

A numerical example of the one-to-one reduction model is given below.

Example: $E(B) = 1.0$, $E(R) = E(T) = .5$, R, B, and T are exponentially distributed. $C_R = $ Rs. 200, $C_M = $ Rs. 100 and $C_I = $ Rs. 1.

A graph of $a(\varepsilon)$ versus the arrival rate λ for different values of a are given in Figure 1. When λ is small the level of opportune maintenance is so high that no failure of the machine occurs. However, when λ is high the reduction in failure rate is not very high.

The plot of $TC_{\infty} - TC_1$ versus the arrival rate λ for different values of a are given in Figure 2. When λ and a are high, opportune maintenance is very beneficial. However, when λ and a are low, opportune maintenance increases the cost of operation. This is mainly because the rate of opportune maintenance is high and because zero failure rate can be achieved with lower opportune maintenance rate. Therefore it seems that it is not necessary to carry out opportune maintenance every time the machine begins idling. Maybe the opportune maintenance should be carried out every rth time the machine begins idling. Then using the properties of alternating regenerative processes[7] it can be shown that the mean inventory $E(N)$ and mean opportune maintenance rate θ_r for the above model is given by

$$E(N_r) = \lambda E(S_r) + \frac{\lambda^2 E(S_r^2)}{2[1-\lambda E(S_r)]} + \frac{\lambda^2 E(T^2)}{2[(r-1)+\lambda E(T)+\tilde{T}(\lambda)]}$$

and

$$\varepsilon_r = \frac{\lambda[1-\lambda E(S_r)]}{\lambda E(T)+\tilde{T}(\lambda)+r-1}, \qquad (12)$$

where

$$E(S_r) = [1+\alpha(\varepsilon_r)E(R)]E(B)$$

and

$$E(S_r^2) = \alpha(\varepsilon_r)E(B)E(R^2) + [1+\alpha(\varepsilon_r)E(R)]^2 E(B^2)$$

The total cost per day under this policy is

$$TC_r = E(N_r)C_I + \lambda\alpha(\varepsilon_r)E(B)C_R + \varepsilon_r C_M \qquad (13)$$

Note that when $r=1$, $TC_r = TC_1$ is given by (8) and as $r \to \infty$, $TC_r = TC_8$ is given by (5). Now one can choose r such that TC_r is minimized.

GRAPH OF α(ε) VS λ FOR DIFFERENT VALUES OF α

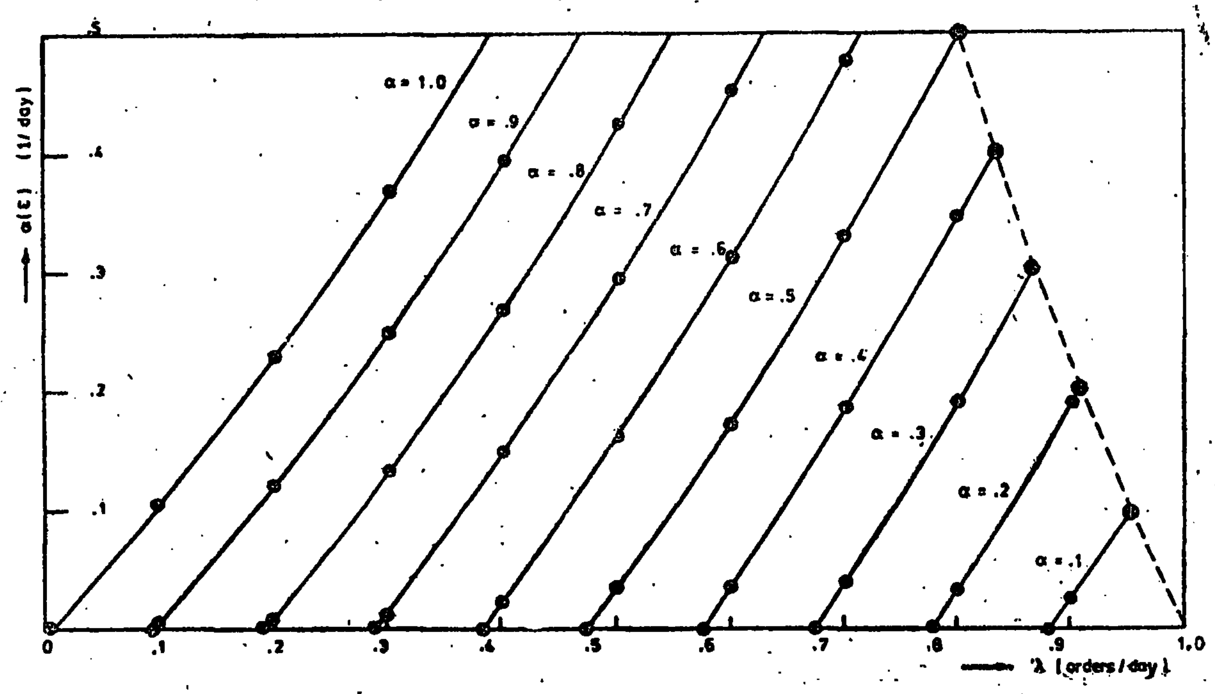

Fig. 1

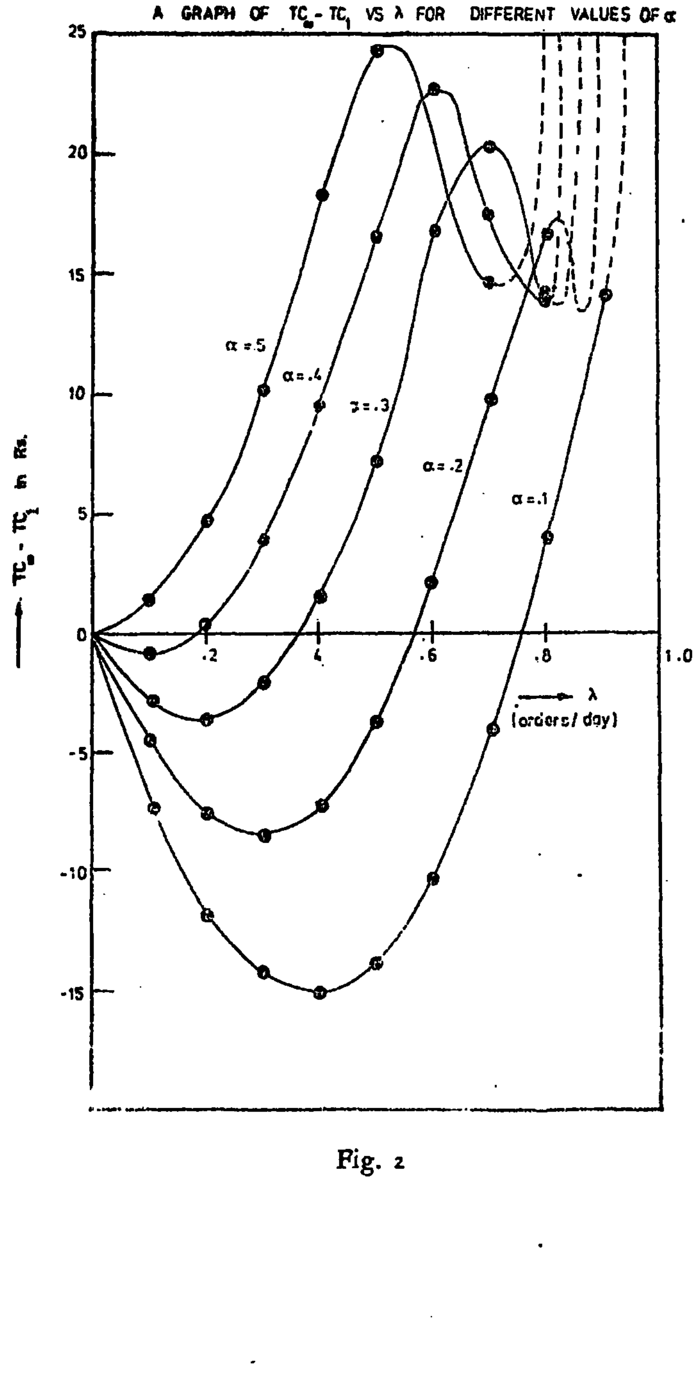

Fig. 2

Conclusion

In this paper we have introduced the notion of opportune maintenance and its effects in a single machine job shop are discussed. The job shop without opportune maintenance was modelled as an M/G/1 queue and the job shop with opportune maintenance as an M/G/1 queue with single 'server vacations'. Using the results available for these models the mean inventory in the job shop and cost rates were derived with and without opportune maintenance. Through an example it was illustrated that opportune maintenance carried out every time the machine begins idling is not always beneficial. Then as an alternative it was proposed that the opportune maintenance should be carried every rth time the machine begins idling and the cost rate for this policy is derived. Using this cost rate the optimal value for r may be evaluated.

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