OPTIMAL LOAD FLOW STUDY ON THE 1984 POWER SYSTEM OF SRI LANKA

by

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Introduction

In the present day planning and operation of a Power System, a Power Flow program is an essential requirement. Even a small saving in the Electrical Power generated results in a large financial saving annually, due to the high cost of electrical energy. For the economic operation of a power system, it is thus desirable to have an optimal power flow solution. In the usual Power Flow Computer Program, the electrical power in the network is uniquely defined when all the control parameters are specified. On the other hand, in an optimal power flow program, the control parameters are adjusted in accordance to a given criterion, until the Optimal Power Flow configuration is reached. This is done by defining an objective function which is maximised or minimised to find the optimal solution.

In this study the steepest descent method developed by Dommel and Tinney[1] is adopted to find the Optimal Power Flow solution. The Lagrangian multipliers which have to be calculated in this method are evaluated by the method given by Burchett et al[2] to increase the speed of the program appreciably.

At present, the Sri Lanka Power System is operated intuitively, and not at the optimal solution. Thus the percentage power loss in the system could be rather high. The aim of the paper is to present a program to study the optimisation of the operation of a power system. The method of optimisation is illustrated by considering the 1984 Sri Lanka Power System.

Methodology

The program developed, starts from a non optimal but feasible solution, which is successively improved to give the optimal solution. In this study the active and reactive power flows are optimized separately. The objective functions and control parameters are as follows :

Active Power Flow :

Objective function = Total Operating Cost (1)

Control parameters : Active power at generator busbars

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Reactive Power Flow :

Objective Function = Active Power at slack busbar (2)

Control parameters : Voltage at Generator busbars

Since the concept of the slack busbar is introduced to take the system losses into consideration, the minimization of the active power injection at the slack busbar is identical to minimization of the system losses.

Feasible Power Flow Solution

Each busbar in a power system is characterised by four variables P_{Net}, Q_{Net}, V and θ of which two are specified and the other two are determined by the Power Flow Program. In such a program the busbars are categorized into three types namely,

i. Slack busbar (One of the generator nodes): V, θ specified; P_{Net} and Q_{Net} to be found.

ii. P-V busbars (usually generator nodes): P_{Net}, V specified; θ and Q_{Net} to be found.

iii. P-Q busbars (usually load busbars): P_{Net}, Q_{Net} specified; V and θ to be found.

For a N busbar system the power flow is governed by equations (3) for each node i

$$P_{Net,i} + jQ_{Net,i} = V_i e^{j\theta_i} \sum_{k=1}^{N} (G_{ik}-B_{ik})V_k e^{-j\theta_k} \quad (3)$$

With initial estimates for the unknown variables, equation (3) is not exactly satisfied. Thus if the calculated values of the right hand side of equation (3) is written in the form

$$P_i(V,\theta) + jQ_i(V,\theta)$$

then the active and reactive power mismatches can be written as

$$\Delta P_i = P_{Net,i} - P_i(V,\theta) \quad (4)$$

$$\Delta Q_i = Q_{Net,i} - Q_i(V,\theta) \quad (5)$$

At any feasible solution $\Delta P_i = 0$ and $\Delta Q_i = 0$

Equations (4) and (5) may be combined and expressed in the form

$$[g(x,y)] = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

where [x] is the vector of unknown θ of P-V and P-Q busbars and unknown V of P-Q busbars.

[y] is the vector of specified variables (i.e V and θ of slack busbar, P_{Net} and V of P-V busbars and P and Q of P-Q busbar).

Also the power mismatch vectors are [ΔP] for P-V and P-Q busbars, and [ΔP] for P-Q busbars.

The Jacobian matrix of partial derivatives required for the Newton-Raphson algorithm is given by

$$\left[\frac{\partial g(x,y)}{\partial x}\right] = \begin{bmatrix} \frac{\partial \Delta P}{\partial \theta} & \frac{\partial \Delta P}{\partial V} \\ \frac{\partial \Delta Q}{\partial \theta} & \frac{\partial \Delta Q}{\Delta V} \end{bmatrix} \quad (7)$$

the incremental vector for updating [x] may be written as

$$[\Delta x] = -\left[\frac{\partial g9x,y)}{\partial x}\right]^{-1} [g(x,y)] \quad (8)$$

For the sake of symmetry of structure, the Jacobian is modified, and the mismatch equation (6) is re-written in the form

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \frac{\Delta V}{V} \end{bmatrix} \quad (9)$$

where

$$H_{ik} = \frac{\partial P_i(V,\theta)}{\partial \theta_k} \qquad N_{ik} = V_k \frac{\partial P_i(V,\theta)}{\partial V_k} \quad (10)$$

$$J_{ik} = \frac{\partial Q_i(V,\theta)}{\partial \theta} \qquad L_{ik} = V_k \frac{\partial Q_i(V,\theta)}{\partial V_k}$$

It has been shown[3] that it is possible to modify equation (9) to two sets of decoupled equations

$$\begin{bmatrix} \frac{P}{V} \end{bmatrix} \doteq [B'] \, [\Delta\theta] \qquad (11)$$

$$\begin{bmatrix} \frac{Q}{V} \end{bmatrix} = [B''] \, [\Delta V] \qquad (12)$$

where $[B']$ and $[B'']$ are constant matrices.

The evaluation of $[\Delta\theta]$ & $[\Delta V]$ by the use of equations (11) and (12) are known as the fast decoupled Newton Raphson algorithm.

The advantage of this modification is that it increases the speed of computations appreciably since both $[B']$ and $[B'']$ are constants. In the solution of the two sets of simultaneous equations, triangular factorization is used. This is a modification of the standard Gaussian elimination method, where repeated factorization of $[B']$ and $[B'']$ is avoided. The values of $[\Delta\theta]$ and $[\Delta V]$ are found by forward backward substitution, which is a short procedure, to increase the speed.

In a practical power system, most matrices of this program would be sparse (i.e. most elements are zeros). This property is exploited to minimize the computer storage and increase the speed of computation.

The algorithm for the load flow program and the sparsity technique was adopted from a paper by S. Karunaratne et al [3]

Optimal Power Flow Solution

Unlike in the usual power flow solution the specified parameters are no longer considered completely fixed. Some of these parameters form the control parameters (u) in the optimal solution, while the remaining parameters (p) are kept fixed.

Thus equation (6) becomes the equality constraints

$$[g(x,u,p)] = 0 \qquad (13)$$

The objective function defined in equation (1) and (2) may be expressed for active and reactive power as follows.

For active power

$$f_{act} = \sum_{i=i}^{N_g} C_i . P_{G,i} \qquad (14)$$

The slack bus has to be considered in this function. If no cost were associated with the power of the slack busbar, the minimization process would try to assign all the power to the slack busbar.

As $P_{G,1}$ is not an independent variable, it is given in the following manner

$$P_{G,1} = P_1(V,\theta) + P_{L,1}$$

where $P_{L,1}$ - load at the slack busbar.

Thus

$$f_{act} = C_1.(P_1(V,\theta) + P_{L,1}) + \sum_{i=2}^{N_g} C_i . P_{G,i} \qquad (15)$$

For reactive power

$$f_{react} = P_1(V,\theta) \qquad (16)$$

Thus we need to minimize a function $f\,(f_{act}, f_{react})$ subject to the constraints $[g(x,u,p)] = 0$

Since f_{act} and f_{react} are functions of x and u, the function to be minimised may be written as f(u,x). Thus from classical optimization theory, we may find the minimum by defining an unconstrained Lagrangian function. Thus

$$[L(x,u,p)] = [f(x,u)] + [\lambda]^T \, [g(x,u,p)] \qquad (17)$$

Where $[\lambda]$ is the vector of Lagrangian multipliers.

The necessary set of conditions for a minimum are given by equations (18), (19) and (20)

$$\left[\frac{\partial L}{\partial x}\right] = \left[\frac{\partial f}{\partial x}\right] + \left[\frac{\partial g}{\partial x}\right]^T [\lambda] = 0 \qquad (18)$$

$$\left[\frac{\partial L}{\partial u}\right] = \left[\frac{\partial f}{\partial u}\right] + \left[\frac{\partial g}{\partial u}\right]^T [\lambda] = 0 \qquad (19)$$

$$\left[\frac{\partial L}{\partial \lambda}\right] = [g(x,u,p)] = 0 \qquad (20)$$

where $\left[\frac{\partial g}{\partial x}\right]$ is infact the Jacobian matrix of the power flow program.

<u>Algorithm for the Optimal Power Flow Solution</u>

The steps of the algorithm may be stated as follows.

Step (1) - Assume an initial set of control parameters [u]

Step (2) - Solve for [x] by solving the Power Flow problem using the fast decoupled Newton Raphson algorithm.

Step (3) - Solve for the Lagrangian Multipliers from

$$[\lambda] = -\left[\frac{\partial g}{\partial x}\right]^{T^{-1}} \left[\frac{\partial f}{\partial x}\right] \qquad (21)$$

where $\left[\frac{\partial f}{\partial x}\right] = C_1 \cdot \begin{bmatrix} H_1 \\ N_1/V \end{bmatrix}$ for active power flow,

$$H_1 = \left[\frac{\partial P_1(V,\theta)}{\partial \theta}\right]$$

$$N_1 = \left[\frac{V\, \partial P_1(V,\theta)}{\partial V}\right]$$

and $\left[\frac{\partial f}{\partial x}\right] = \begin{bmatrix} H_1 \\ N_1/V \end{bmatrix}$ for reactive power.

Then equation (21) becomes

$$\begin{bmatrix} \lambda p \\ \lambda q \end{bmatrix} = -z \begin{bmatrix} H & N \\ J & L \end{bmatrix}^{T-1} \begin{bmatrix} H_1 \\ N_1 \end{bmatrix} \qquad (22)$$

where $z = C_1$ in active power flow or $z = 1$ in reactive power flow and $[\lambda]$ is expressed in the form

$$[\lambda] = \begin{bmatrix} \lambda_p \\ \lambda_q \end{bmatrix}$$

The Lagrangian multipliers can be calculated using equation (22). The approximated Jacobian used in the fast decoupled Newton Raphson Algorithm does not yield sufficiently accurate results. Thus the Jacobian matrix should be evaluated and factorized in each optimal power flow equation. However, this is time consuming, so that the problem is solved using the method suggested by Burchett et al[2]. It has been found that by using this method the Lagrangian multipliers can be obtained by a slight modification to the Load flow program using the fast decoupled Newton-Raphson algorithm. The Lagrangian multipliers are evaluated by using the following equations (23) to (26) iteratively.

$$\left[\frac{\Delta 1_Q}{V}\right] = [B'] \, [V \Delta \lambda_p] \qquad (23)$$

$$\left[\frac{\Delta 1_V}{V}\right] = [B''] \, [V \Delta \lambda_q] \qquad (24)$$

$$[\Delta 1_\theta] = \left[\frac{\partial f}{\partial \theta}\right] - \left[\frac{\partial P}{\partial \theta}\right]^T [\lambda_p] - \left[\frac{\partial Q}{\partial \theta}\right]^T [\lambda_q] \qquad (25)$$

$$[\Delta 1_V] = V\left[\frac{\partial f}{\partial V}\right] - \left[\frac{V \partial P}{\partial V}\right]^T [\lambda_p] - \left[\frac{V \partial Q}{\partial V}\right]^T [\lambda_q] \qquad (26)$$

$[\lambda_p]$ and $[\lambda_q]$ assumed initially then $[\Delta 1_\theta]$ and $[\Delta 1_v]$ are evaluated by equation (23) and (24) and $[\lambda_p^{old}]$ and $[\lambda_q^{old}]$ is improved to

$$[\lambda_q^{new}] = [\lambda_p^{old}] + [\Delta \lambda_p]$$

$$[\lambda_q^{new}] = [\lambda_q^{old}] + [\Delta\lambda_q]$$

This procedure is repeated until $[\Delta l_\theta]$ and $[\Delta l_v]$ are sufficiently small.

When $[\Delta l_\theta]$ and $[\Delta l_v]$ are zero, it is seen that the Lagrangian multipliers satisfy equation (22).

This procedure is very similar to the calculation of $[\Delta\theta]$ and $[\Delta V]$ in the fast decoupled Newton Raphson algorithm. Since [B'] and [B"] are the same matrices used in the load flow program re-factorization of these matrices are, not necessary, as this has been done in the load flow program. In fact the same procedures used in the load flow program can be used to evaluate the Lagrangian multipliers. Due to this reason the results can be obtained quickly.

Step (4) - Compute the gradient vector from

$$[\nabla f] = \left[\frac{\partial f}{\partial \bar{u}}\right] + \left[\frac{\partial g}{\partial \bar{u}}\right]^T$$

or $$[\nabla f] = \left[\frac{\partial f}{\partial \bar{u}}\right] + \begin{bmatrix} \frac{\partial P}{\partial u} \\ \frac{\partial Q}{\partial u} \end{bmatrix}^T \begin{bmatrix} \lambda p \\ \lambda q \end{bmatrix}$$

In the case of optimal active power flow, the control parameters are P's of generator busbars and in the case of optimal reactive power flow the control parameters are the voltages at generator busbars.

Thus it is seen that gradient with respect to each control parameter is given by

$$\frac{\delta f_{act}}{\delta P_{Gi}} = C_i - \lambda_{pi}$$ for active power flow.

$$\frac{f_{react}}{V_i} = N_{1i} + \sum \lambda_{pk} N_{ki} + \sum \lambda_{qk} L_{ki}$$

for reactive power flow.

Step (5) - Check for convergence.

If $[\nabla f]$ is sufficiently small then the optimal solution has been reached else go to step 6.

Step (6) - Computer step length from

$$[U] = d.[\nabla f]$$

Where the constant d has to be assumed carefully. If d is too small the number of iterations to reach the solution will be large, and if d is too large the program will circle the solution.

Step (7) - Advance the control parameters

$$\left[U^{New}\right] = \left[U^{Old}\right] + \left[\Delta U\right]$$

Step (8) - Go to step (2)

Results

A shematic diagram of the system used to illustrate the optimal power flow program is shown in figure 1. This is a particular configuration of the 1984 Sri Lanka Power System. This example describes the optimization of the active and reactive power flow. The reduction of the operating cost and the system losses are shown in table 1 and 2 respectively.

TABLE 1

Reduction in operating cost

Iteration number	Cost/Unit time
1	43.004
2	41.376
3	41.311
4	41.257
5	41.220
6	41.215

TABLE 2

Reduction in System losses

Iteration	System	% System losses
1	0.330	7.231
2	0.296	6.420
3	0.269	5.984
4	0.258	5.750
5	0.250	5.585
6	0.245	5.465

Conclusions

Both the system losses and the operating cost is reduced by this program. The operating cost is reduced by around 5% and the system losses are reduced by 8%. Thus this program yields a fast solution to the optimal power flow problem. Since the cost of Electrical Energy is high this reduction in operating cost and system loss will result in a considerable financial saving.

References

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3. Prof. S. Karunaratne, Dr. J.R. Lucas, Dr. H. Sriyananda, Mr. A.S. Induruwa, Mr. B.S.P. Perera: A fast decoupled Sparsity Oriented algorithm for the Load Flow problem in Electric Power Systems : Transactions of the Institution of Engineers, Sri Lanka, 1976.

List of Symbols

$P_{NET,i}$, $Q_{NET,i}$ - Active power and reactive entering node i

V_i - Voltage magnitude at node i

θ_i - Voltage angle at node i

$G_{ik} + jB_{ik}$ - (i,k)th element of nodal admittance matrix

P_{Gi}, Q_{Gi} - Active power and reactive power generation at node i

P_{Li}, Q_{Li} - Active power and reactive power loads at node i

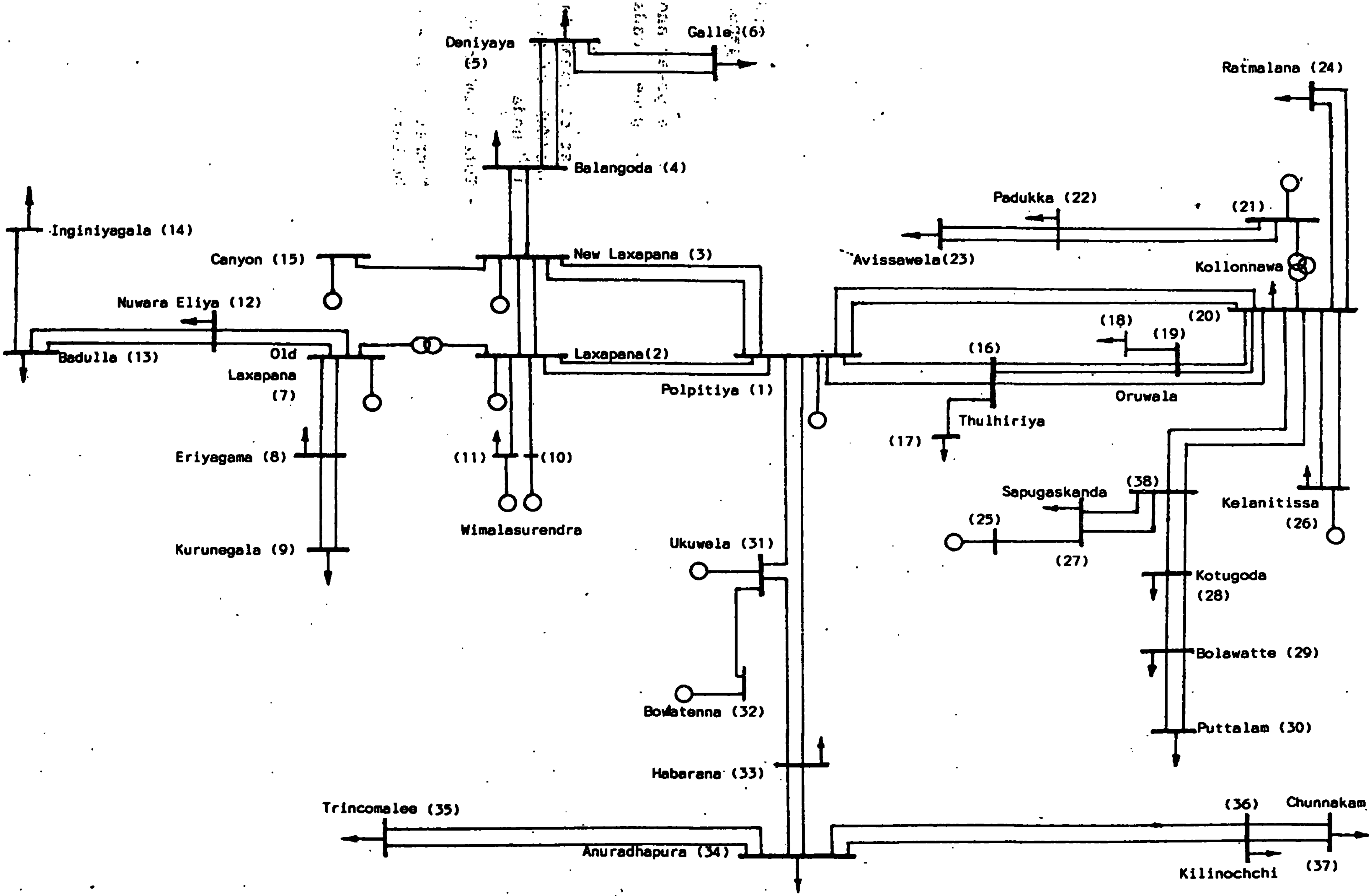
C_i - Cost of generation (Rs/Mwh) of generator at i^{th} node

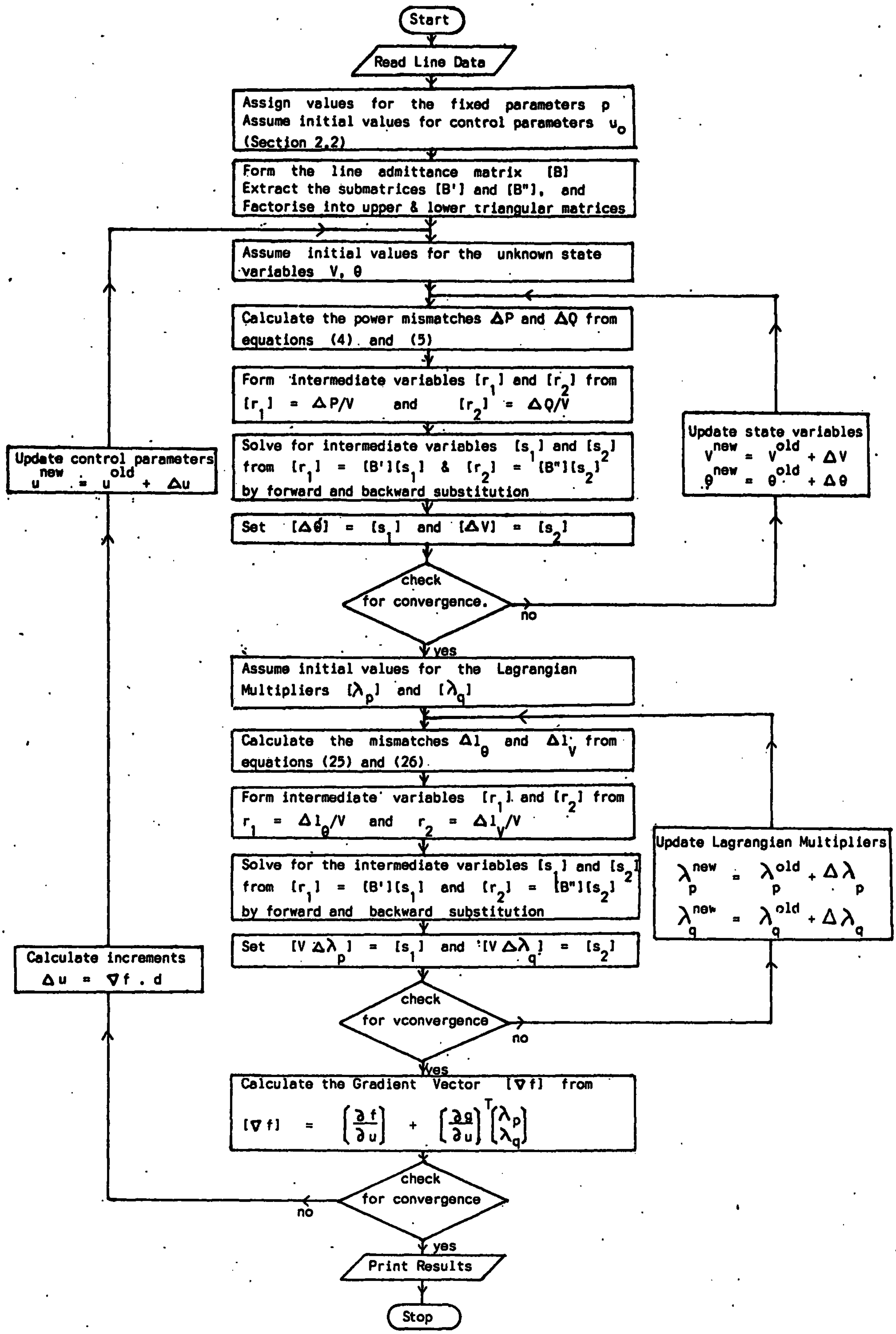
N - Total number of busbars

N_g - Number of generator busbars

FIGURE 2

SRI LANKA POWER SYSTEM - 1984





FLOW CHART OF COMPUTER PROGRAM