SYNCHRONOUS MACHINE TRANSIENT STABILITY ANALYSIS BY AN EXTENSION TO THE POINT BY POINT METHOD OF SOLUTION OF THE SWING EQUATION

by

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Introduction

The analysis of a synchronous machine is described by a set of differential equations. The number of differential equations required for a single machine depends on the model required to represent accurately, the machine performance. The swing equation which describes the motion of the machine rotor, is a second order differential equation and is the simplest representation of the synchronous machine. In order to determine the angular displacement between the machines of a power system during transient conditions, it is necessary to solve the swing equations of the machines involved. The solution provides necessary details to determine the individual stability of the machines and thus the transient stability of the power system.

The swing equation which describes the variation of load angle versus time, could be solved for the duration concerned, (this being around one second in transient stability studies) by using several methods available for solution of ordinary differential equations. Modified Euler, Runga-Kutta, and point by point methods have been widely applied for the solution of the swing equation in transient stability studies. Unlike the other two, the point by method developed by Dahl in 1938, is mostly used to solve this equation in simple cases, where hand calculations are involved. Because of its simplicity, this is the preferred method used in class rooms for the teaching of the transient stability problem.

Runga-Kutta and modified Euler, methods of solution of ordinary differential equations, could be used to solve the swing equation even in the presence of damping, but the use of the existing point by point method has been restricted by the fact that the swing equation should not include the damping component.

If a machine under transient conditions experiences an oscillatory variation in load angle, it is considered to be stable and if the load angle increases continuously, it will be unstable. The oscillatory variation in load angle obtained for an undamped machine is not very helpful to a lecturer to convince the students that the machine under consideration is stable. In practice, there is a finite damping factor in every machine and this should be included in the swing equation. When the swing equation is solved for the conditions where an undamped machine is stable, the load angle variation obtained when damping is included, is a damped oscillation, and finally the load angle converges to a stable value. Therefore the inclusion of the damping component in the

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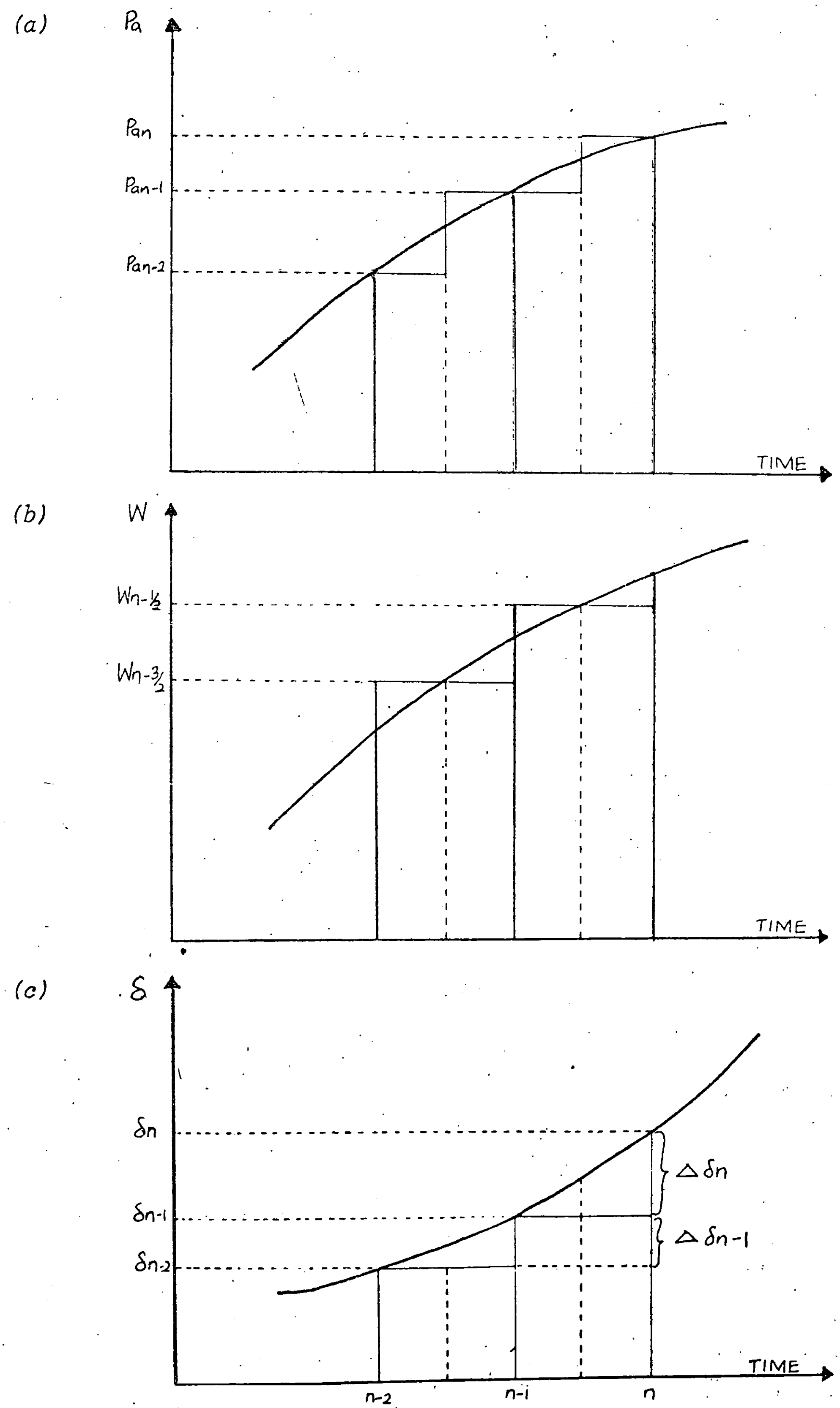
(a)
Pa
Pan
Pan-1
Pan-2
TIME
(b)
W
Wn-½
Wn-3/2
TIME
(c)
δ
δn
δn-1
δn-2
Δδn
Δδn-1
TIME
n-2
n-1
n

Figure 1

swing equation is very important because it represents a more realistic case. The existing point by point method is not capable of solving this swing equation in the presence of damping and hence an extension to this method has been formulated in this paper which is very useful especially in classroom work.

List of Symbols -

P_a	- Accelerating Power
P_{an}	- Accelerating power at the end of the n^{th} time interval
h	- time interval
M	- M constant of the machine
K	- Damping coefficient
δ	- Load angle
δ_n	- Load angle at the end of n^{th} time interval
X_1	- Prefault transfer reactance
X_2	- Post fault transfer reactance
P_s	- Shaft power
E	- interval emf of the machine
$W_{n-\frac{1}{2}}$	- Rate of change of load angle at the middle of the n^{th} time interval
λ	- Modification factor for damping = Kh/2M

Difference Equation for the Existing Point by Point Method

The swing equation of any machine, without damping, could be expressed in the form

$$M.\frac{d^2\delta}{dt^2} = P_a \qquad (1)$$

With reference to Fig. 1, it is clear that

$$W_{n-1/2} - W_{n-3/2} = \frac{d^2\delta}{dt^2} . h \qquad (2)$$

From equations (1), (2) an with reference to Fig. 1, con ideing the appropriate interval of t me, the following equation co d e written.

$$W_{n-1/2} - W_{n-3/2} = \frac{P_{an-1}}{M} . h \qquad (3)$$

Also from figure 1b and 1c, the following equations can be derived.

$$\delta_{n-1} - \delta_{n-2} = W_{n-3/2} . h \qquad (4a)$$

i.e. $\Delta\delta_{n-1} = W_{n-3/2} . h$

$$\delta_n - \delta_{n-1} = W_{n-1/2} . h \qquad (4b)$$

i.e. $\Delta\delta_n = W_{n-1/2} . h$

From equations (4) and (3) we get

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{P_{an-1}}{M} . h^2 \qquad (5)$$

Equation (5) represents the difference equation written for the existing point method for transient stability studies.

Extension to the Point by Point Method

The swing equation with the damping component could be expressed in the form,

$$M.\frac{d^2\delta}{dt^2} + K.\frac{d\delta}{dt} = P_a \qquad (6)$$

The equation (6) could be rewritten as

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} - \frac{K}{M}\frac{d\delta}{dt} \qquad (7)$$

From equation (2) & (7) and considering the appropriate time interval, the following equation is obtained

$$W_{n-1/2} - W_{n-3/2} = \frac{P_{an-1}}{M} . h - \frac{K}{M}\left[\frac{d\delta}{dt}\right]_{n-1} . h \qquad (8)$$

$$\text{But } \left[\frac{d\delta}{dt}\right]_{n-1} = \frac{W_{n-1/2} + W_{n-3/2}}{2} \qquad (9)$$

From (4), (8) and (9) we get the equation,

$$W_{n-1/2} - W_{n-3/2} = \frac{P_{an-1}}{M} . h - \frac{\lambda}{h}(\Delta\delta_{n-1} + \Delta\delta_n) \qquad (10)$$

From (4) and (10), the following equation is derived,

$$\Delta\delta_n = \frac{(1-\lambda)}{(1+\lambda)} \cdot \Delta\delta_{n-1} + \frac{1}{(1+\lambda)} \frac{P_{an-1} \cdot h^2}{M} \quad (11)$$

This equation (11) represents the difference equation which has been derived for the calculation of load angle of a machine with a finite damping coefficient, under transient conditions.

The constant λ in equation (11) is possitive and is equal to (K.h)/2M. The coefficients of $\Delta\delta_{n-1}$ and of $(P_{an-1}/M)h^2$ in equation (11) are different from those of equation (5). Those coefficients in (5) are equal to unity but in (11), these are less than unity. This indicates that the increase in load angle has been decreased by the damping effect of the machine and the amount of reduction is determined by the factor λ which is directly related to the damping coefficient.

The validity of the equation (11) has been verified by comparing the results with those obtained from Runga-Kutta (4th order) method. The error term of this difference equation is of the order of h^4 and therefore the method is accurate enough for transient stability studies. By selecting a smaller step size (h value) the accuracy of the results could be improved.

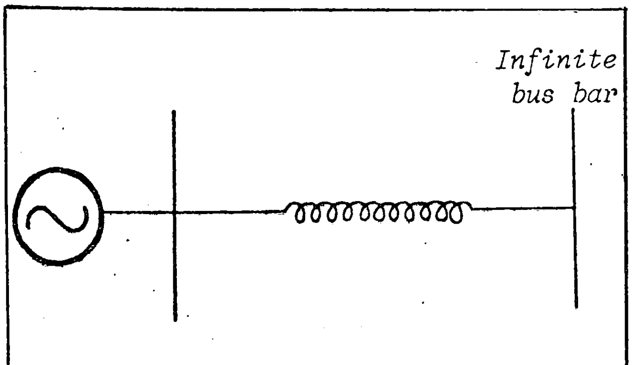

Figure 2

Typical Cases

Example 1

The system under consideration is shown in Figure (2). A synchronous machine is connected to an infinite bus bar with a transmission line and as a result of a fault in the system at time zero, the reactance changes.

The system data are as given below:

X_1 = 0.6 pu

X_2 = 0.9 pu

E = 1.133 pu

V = 1.0 pu

M = 0.0127 sec²/radian

h = 0.01 sec.

Per unit quantities are based on machine rating of 40 MVA.

The variation of load angle without damping component is oscillatory as shown in curve (a) of Figure (3). Introducing a damping coefficient K of 5.08×10^{-3}, (λ = 0.002), the machine swing equation is solved for load angle using Extended Point by Point and Runga-Kutta methods.

The variations of load angle obtained from these two methods are shown in curves (b) and (c) of Fig. 3 which are almost coincident. These show a damped oscillation of the load angle which finally converges to a stable value.

Example 2

In this example, the post fault line reactance, X_1, has been assumed to increase to 1.28 pu while keeping other data at the values given in the previous example.

When the swing equation of the machine

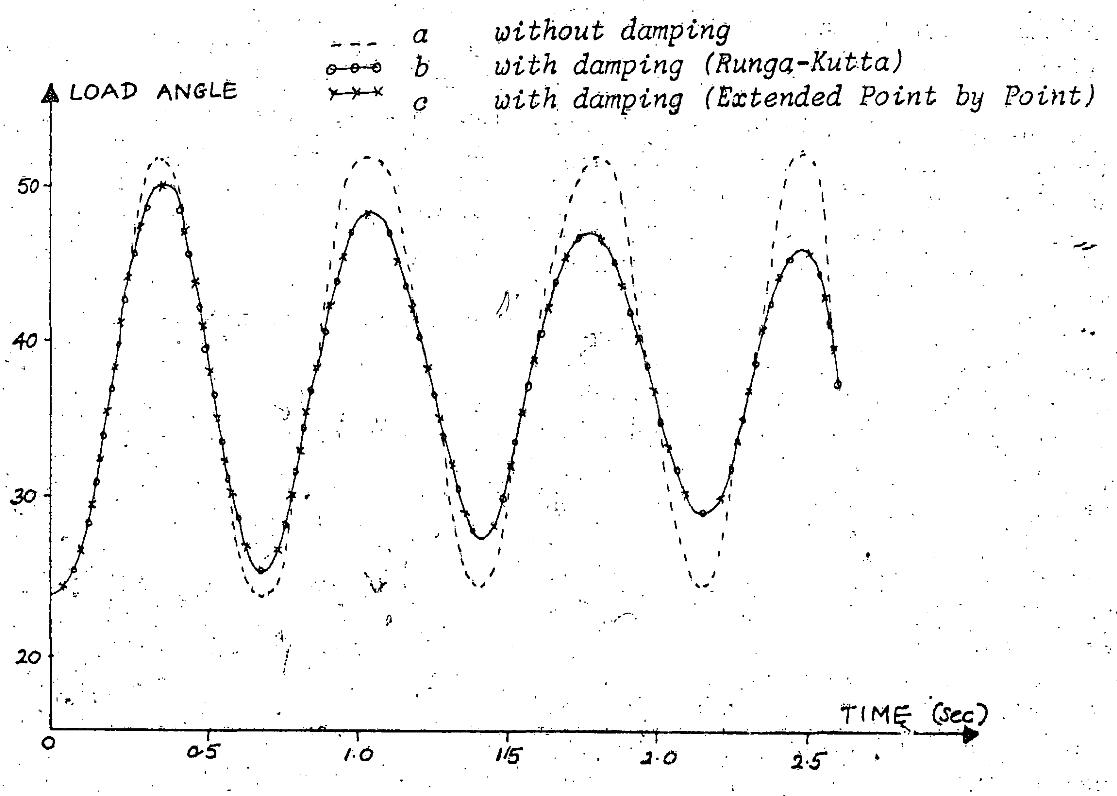

Figure 3 - Load angle variation with time

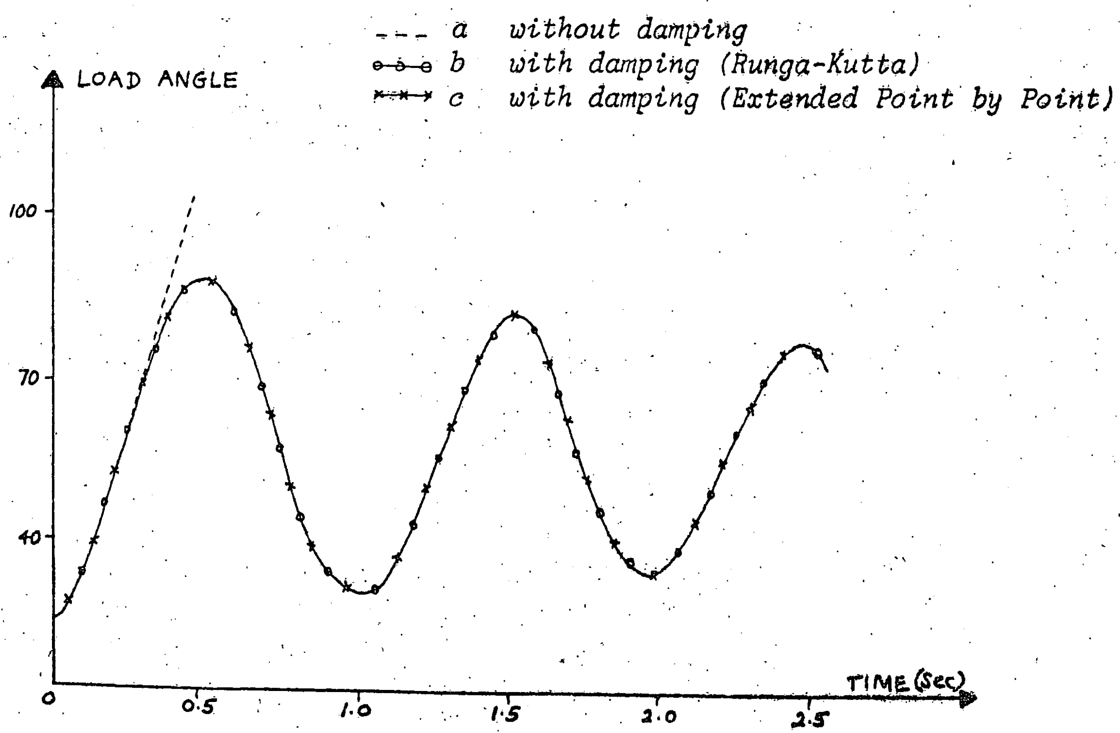

Figure 4 - Load angle variation with time

solved neglecting the damping effect, the variation of load angle a steep near parabolic increase shown in curve (a) of, Fig. 4 which indicates an unstable condition. the inclusion of damping effect a damping coefficient K of x 10^{-3}, (λ = 0.002) causes load angle variation to be a damped oscillation as shown in curve and (c) of Figure 4 which shows stable variation. The curves and (c) of Fig. 4 obtained by Runga-Kutta method and the Extended Point by Point Method (as propounded this paper) respectively are not over lapping each other.

Conclusions

example show the importance inclusion of the damping effect the swing equation. When the damping effect is taken into account, would easily be able to differentiate the stable and unstable variation of load angle because they are clearly identified from the results and graphs drawn.

The Extended Point by Point Method of solution of the swing equation with a finite damping factor has been satisfactorily employed in the two examples without an appreciable deviation of the results from those obtained from the Runga-Kutta method. Therefore the Extended Point by Point Method will be very useful in the solution of the transient stability problem, specially in the simpler cases which are mostly used in the teaching of the transient stability problem.

Reference

Glenn W. Stagg and Ahmed H. El-Abiad, "Computer Methods in Power System Analysis".