

MULTI CRITERIA PRODUCTIVITY MEASUREMENT AND DECISION MODEL

by

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Abstract

The currently available productivity measurement models could be broadly categorised as Quantitative and Qualitative. Each category suffers from inherent limitations. The approach suggested is to effectively combine the strengths of Quantitative and Qualitative models and evolve a hybrid approach. The hybrid model is then taken and optimised using non-linear Goal programming in order to generate future target figures for machine failure rates and Effective Utilisation of plant and machinery.

1.0 Introduction and Literature Survey

The current Productivity measurement models could be broadly categorised as Quantitative and Qualitative. The Quantitative models are based on input-output indices and suffer from the following inherent limitations:

- They do not measure changes in Productivity against some norm or standard. For example, if while measuring the work the method changes, the norm also changes.
- They do not help in choosing optimal inputs so that the cost of operations are minimised and Productivity is optimised.
- They do not consider the causal relationships amongst the subsystems in an organisation (e.g: Technical subsystem, Financial subsystem, Commercial subsystem, Marketing subsystem ... etc) [1]
- Different organisations, as different systems and subsystems, can have different goal seeking properties. Productivity models established on defined premises, do not always incorporate and measure these goals but survey shows that such models measure only the ratio of some output to some input for a particular time period. They are thus strategically deficient.

There had been attempts made by various researchers to overcome some of the limitations referred to above. Two profit linked total factor Productivity measurement models have been developed and are referred to as the APC model [or multiplicative

model] [2] [3] and the PPP model [Additive model] [2]. These profit linked total factor Productivity models are strategy sufficient and the causal impact of sub-systems could be reasonably depicted. They however, suffer from other limitations of which the inability to measure the contribution of intangible factors to the improvement of Productivity is a major one.

Several Qualitative models have been developed in the recent past [4] [5]. These Qualitative models cannot be considered as strict Productivity measurement models and often an inflationary riddled overall system performance measure (eg. gross profit, return on capital employed etc.) is related to inflationary free intangible factors such as Productivity, Effectiveness etc.

The approach suggested is to effectively combine the strengths of Quantitative and Qualitative methods and evolve a hybrid technique. The model developed is then utilised as an optimisation or deterministic tool thus eliminating one of the inherent weakness of most Productivity measurement model.

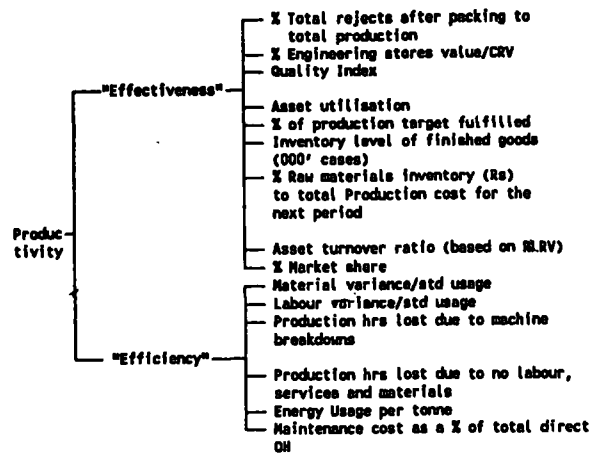
The model is validated using data from a large process based multi national company and historical data from January 1991 to December 1993 is utilised. The historical data collected are for tangible variables shown in Fig 1.0.

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Note: CRV = Current Replacement Value of Plant and Machinery
 RLRV = Remainder Life Replacement Value of Plant and Machinery
 OH = Over Head Cost

Figure 1 - Tangible Variables Defining the Criteria

2.0 The Productivity Measurement Model

Productivity is defined Qualitatively as how 'Effectively' and 'Efficiently' input resources are used to produce outputs. 'Effectiveness' has been further defined as the level of achievement in fulfilling the demands of external systems[4]. Several tangible variables, from different sub-systems in the Organisation are chosen to define the 'Effectiveness' criteria. Similarly, 'Efficiency' is defined as the level of achieving results in relation to internal resources consumed [4] and tangible variables from different subsystems are chosen to define the criteria. Fig 1.0 depicts the tangible variables defining the Qualitative criteria and its relationship to Productivity. The tangible variables chosen are specific to a process based industry and have been chosen by considering the critical demands of external systems on the production system eg: the Commercial system, Financial system, Marketing systemetc., and demands on internal resources consumed.

The tangible variables are reduced to unitless scores using the Delphi process and the Utility function curves. Questionnaires were circulated to a group of 20 Senior Managers of the Company (ie., 4 from each Functional Department) and their response was obtained. Each variable in the questionnaire consists of a set of attainment levels and the respondents were requested to give grades for different levels of

attainment (ie., a = very poor, b = poor, c = satisfactory, d = good and e = excellent). The questionnaire were then collected, the grades given ranked and the Rank correlation computed. If a convergence is reached, the appropriate Utility function curve is fitted using Orthogonal Polynomials else another round of questionnaire are circulated with each Manager knowing the grades given by their colleagues. This process, known as Delphi technique, was continued until a satisfactory degree of convergence was reached. The Utility function of a variable converts the ratios, or relative values of the variable to a dimensionless (or unitless) values in a common reference scale, spread in a suitable range developed by the user. Each level of a variable in a given period has its corresponding value in a common reference scale referred to as the "achievement score" of the variable [4] [6] [7]. (see Table 1.0 for the equations of the Utility function curves, Fig 2.0 for a sample of graphical representation of the curves and Table 2.0 for historical tangible variables in unitless scores).

Productivity values from January 1991 to December 1993, on a monthly basis, was evaluated using the APC model [2] [3]. The APC model relates Productivity to the overall objective of profitability using the relationship of :

$$\text{Profitability} = \text{Productivity} \times \text{Price Recovery}$$

Table 1 - Equations of Utility Function Curves

Variable	AO	Coefficient of X	Coefficient of X ²	Coefficient of X ³	Coefficient of X ⁴	Coefficient of X ⁵	Coefficient of X ⁶
<u>Technical Production</u>							
% total rejects/Total Prod	99.944	-4320.035	-680768.671	81355025.641	-3144123076.923	41257025641.021	
Quality Index	1323.664	-74.040	1.485	-0.013			
% material Variance/std Usage (perfume)	60.636	-78.430	-5206.978	19699.213	112875.400	-702312.000	956843.399
% material Variance/std Usage (chemicals)	61.858	17.846	-11207.263	26525.533	614335.226	-2661328.507	
% material Variance/std Usage (wrappers)	77.754	-336.308	-34492.084	305581.210	4595211.731	-67432403.021	222124821.283
% material Variance/std Usage (Corr. boxes)	73.877	26.809	-41411.549	-76014.037	9428399.407	11967053.361	-695733254.436
% labour variance/std usage	68.244	-221.845	-400.077	1280.634	3050.035	-3634.298	-6271.396
% no production due to labour, services and material/Available hrs	129.077	-3928.489	114897.046	-2005838.338	17922373.466	-77417216.502	128560627.116
% no production due to machine breakdowns							
Available hrs	113.331	-658.669	-5189.685	87776.201	-461253.092	1070135.505	-931388.235
Asset Utilisation	-54.718	532.217	-1679.246	3744.611	-4205.356	1900.724	
<u>Technical - Engineering</u>							
% Eng. stores value/value of machinery (CRV)	104.659	-958.914	9454.248	-251191.531	1529501.748		
Energy usage per tonne (Steam)	1209.842	-13315.280	65472.165	-169270.156	239414.793	-175997.862	52705.379
Energy usage per tonne (Electricity)	-34107.566	2383.788	-68.295	1.031	-0.009	0.000	0.000
% maintenance cost/Total direct OH cost	112.706	-349.841	335.689	-5621.139	14921.399		
<u>Commercial</u>							
Inventory level of Finished Product ('000 cs) - SL	19.228	13.189	-0.786	0.019	0.000		
Inventory level of Finished Product ('000 cs) - LB	20.014	4616.103	-98857.394	832639.435	-3452226.011	7026215.379	-5614969.183
% raw materials inventory (Rs)/Total Prod. cost	-60.45	3792.119	-78179.431	771236.068	-3531536.882	7406411.039	5792609.321
Production target fulfilled	454.787	-5852.879	28604.66	-67631.286	82531.198	-49516.403	11519.608
<u>Financial</u>							
Asset turnover ratio	-18.465	458.441	-1782.537	5355.856	-9089.39	7666.714	-2493.24
<u>Marketing</u>							
% Market share	83.75	-1214.803	8195.356	-24954.959	39056.532	-30094.992	9037.535

The Final equation would be:

AO	A1 (for x)	A2 (for x ²)	A3 (for x ³)	A4 (for x ⁴)	A5 (for x ⁵)	A6 (for x ⁶)
454.7867	-5852.8789	28604.6601	-67631.2960	82513.1976	-49516.4027	11519.6078

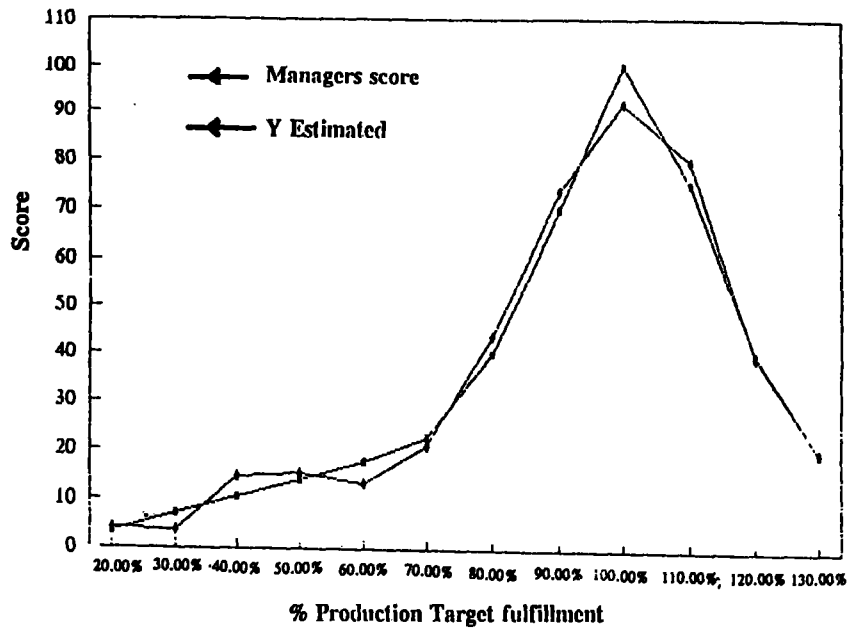


Figure 2 - Graphical Representation of an Utility Function Curves

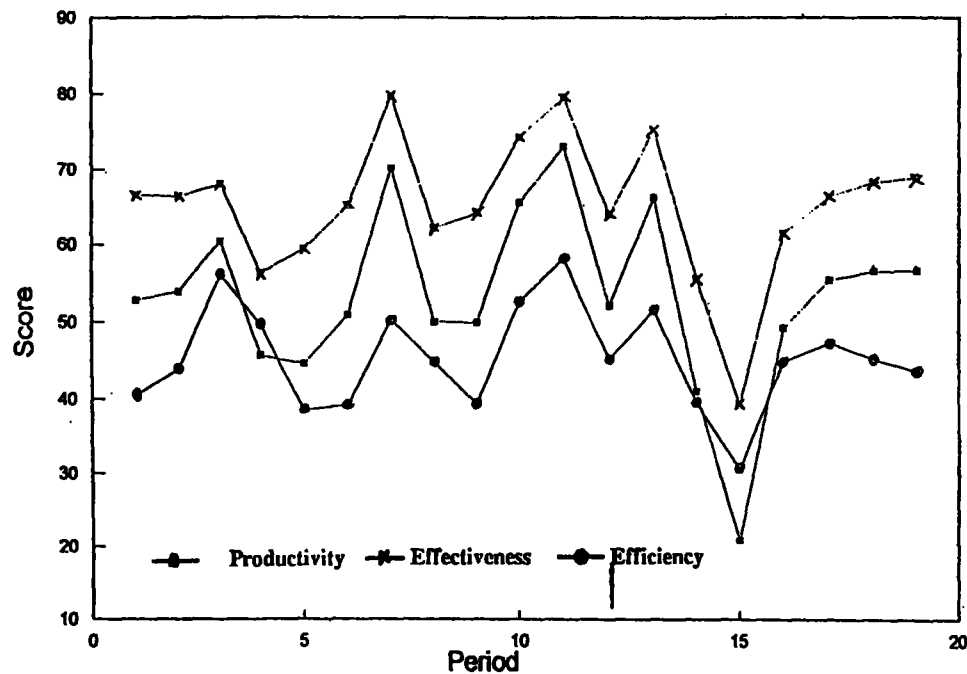


Figure 3 - Trend of Productivity, Effectiveness and Efficiency

Table 2 - Historical Data in Unitless Scores

DATA MATRIX WITH DEPENDENT VARIABLE															
Productivity score	%Total re/ total prod	Quality rejects	%Mat var/std usage	%Labour var /std usage	%No prod due to lab,serv & mat	%No prod due to mach. bkdwns	Asset utilisation	%Eng stores /CRV	Energy usage per tonne	%Maint cost direct OH	InventL of FP:	%Raw mat Inv /Prod cost	Prod. target % fulfilled	%Market share	Asset turn. ratio
JC1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
JC2	17.87883	82.45152	68.03847	12.43912	15.71938	23.87880	27.88443	58.10784	14.10493	78.89284	63.42694	14.82452	19.72013	72.84808	44.81318
JC3	30.49484	68.88648	59.40924	28.03689	32.74680	32.58375	28.98729	50.48744	58.07897	87.78030	19.43236	87.31141	58.23893	71.43177	58.80963
JC4	6.21183	68.38194	56.81310	28.89782	29.68907	31.89028	30.51859	50.39177	38.34447	95.31027	19.43238	88.46530	34.72668	71.77530	58.32859
JC5	10.25179	65.79519	54.44849	26.70386	29.95121	18.90807	24.49083	50.45157	24.20070	83.29503	32.00572	70.73948	54.89920	70.33307	48.82535
JC6	83.98248	68.05350	44.83828	33.19508	62.63815	38.12553	24.57827	48.00981	61.23122	54.35853	34.28104	79.13744	78.10528	70.84872	63.89474
JC7	77.03377	62.73483	47.05247	48.47299	48.24001	47.98923	38.14381	47.30702	64.88692	88.82415	52.53883	29.84858	74.30764	69.11576	69.42883
JC8	10.52378	68.74292	55.34929	12.88620	26.36651	24.38565	21.07348	47.73124	28.20117	64.05382	73.70573	87.85125	14.37178	70.54694	58.99654
JC9	59.89083	68.12281	56.75940	99.71702	23.81286	34.70185	25.63285	47.75548	45.38115	78.39281	74.27845	87.47918	61.74538	70.83283	60.18291
JC10	72.36838	65.36031	59.69869	40.88369	63.81684	28.84588	29.27892	47.70702	50.87498	78.78437	27.99299	31.44454	81.40553	69.05288	68.30286
JC11	51.15153	65.07879	54.55408	54.86599	47.83680	31.27441	30.03878	47.00358	41.05487	92.99698	72.34210	37.83345	63.74928	70.41257	60.14332
JC12	88.25078	68.82891	45.92950	67.44477	34.87974	40.80788	38.63788	47.18568	30.79203	90.90325	72.93859	80.12080	88.03077	72.01192	85.28284
JC13	68.55710	66.40188	54.06873	98.95352	54.06390	42.80388	33.55281	58.57899	58.73890	81.21778	74.52894	54.82168	85.26857	72.78319	72.33273
JC14	24.84227	69.65481	51.51521	60.53592	19.53143	43.72444	28.20881	55.88507	52.35082	78.98165	78.00536	68.55068	44.85835	73.45753	69.14897
JC15	28.82150	68.14263	50.28307	71.98815	48.78180	44.40071	38.37589	58.13072	89.82008	69.78413	74.14939	33.33573	67.20225	74.15477	70.42572
JC16	10.25242	68.79288	47.86241	87.14885	30.03878	33.84005	10.03728	54.90218	40.23210	83.83284	19.43278	68.96231	28.80029	72.11339	84.82700
JC17	54.28885	63.04755	37.98218	67.80651	64.18383	49.51704	33.87017	53.75701	57.91469	69.78413	19.43278	40.75810	75.73821	72.85355	68.57777
JC18	65.16199	64.86786	37.42852	58.75738	68.98323	50.21588	38.82390	53.87971	57.70887	82.83492	19.43278	33.58347	74.25755	71.20815	72.98582
JC19	85.08959	68.08292	58.82230	63.50003	48.49985	57.55195	35.28327	55.50821	58.73242	88.01447	72.78078	28.55881	81.82783	71.30538	78.88461
JC20	58.37037	68.50210	40.57747	58.18157	83.86415	41.89589	34.10438	58.25759	52.18810	88.39428	32.21740	38.14905	87.44357	71.80345	72.20344
JC21	55.43870	64.59601	54.43875	49.45378	48.92091	37.81984	37.78415	55.83038	57.25333	88.91758	72.01593	44.83701	75.33038	71.81887	70.45808
JC22	68.28839	61.70781	55.82031	58.44279	49.45378	40.09737	31.42449	54.97201	48.27825	93.12128	41.57057	51.91301	81.88887	71.44478	77.18200
JC23	57.97697	68.02448	55.85717	52.42805	22.48163	52.31871	34.30480	54.34238	49.37582	85.74639	41.39175	45.71252	75.18887	72.68709	73.01264
JC24	38.86352	68.38541	28.31310	58.84583	19.42415	41.05257	39.41720	49.72078	45.34515	84.94342	58.54482	41.15637	42.84843	71.38148	73.48178
JC25	45.05143	68.37275	47.88341	45.28259	20.30784	42.83978	38.88658	58.25857	44.35234	77.92208	28.31738	32.78877	72.78305	72.00454	74.33088
JC26	47.02064	61.33159	58.77024	44.00820	51.22306	35.75898	37.91132	58.78055	63.24348	70.00838	23.78198	27.16050	73.10324	72.25143	70.43888
JC27	47.32829	67.11804	60.51424	65.90891	60.04813	48.27120	38.20744	59.08530	84.97708	89.08227	38.51108	22.38248	74.47998	75.75855	75.38082
JC28	25.75684	66.81515	87.81270	78.12814	18.48350	38.00580	17.80874	56.72941	56.18448	92.51877	69.58815	70.93348	63.28894	72.23301	77.88828
JC29	38.39813	65.81714	57.06098	51.37537	10.15021	33.75734	32.04838	58.01529	47.80819	77.80131	73.73202	67.48702	65.74974	70.18584	70.50324
JC30	47.81987	61.08223	55.21180	38.73682	27.65748	37.50948	41.92918	55.45984	61.24310	72.18323	65.78650	49.45853	71.15653	73.48275	71.18058
JC31	78.82384	65.90428	65.81884	57.43269	68.43438	32.23247	43.31233	55.89154	64.46953	80.15352	37.02590	13.55492	90.37434	71.75892	79.88330
JC32	55.49165	65.95168	68.28012	57.20450	17.24989	38.08691	38.42636	54.24887	58.43597	81.42387	24.80591	24.14802	65.82383	72.19725	74.83918
JC33	48.29950	66.43449	65.21253	43.33519	63.00785	16.79699	34.02427	52.48510	52.80814	81.68703	83.98854	71.85234	69.03575	73.07282	78.80576
JC34	78.25184	64.13965	58.39058	74.25281	47.97775	37.87224	41.53824	50.28408	70.58892	71.38690	28.37644	38.70539	80.46051	74.11951	85.18352
JC35	71.28970	65.07688	58.86922	88.08452	87.01118	49.87170	42.76375	50.48744	84.41198	83.25381	21.88779	50.99260	80.81920	72.62414	78.02414
JC35	49.17889	64.58837	51.03445	70.70914	20.44573	35.80251	30.77011	51.13198	58.21419	79.99212	4.59669	28.44944	76.36208	74.55903	51.99818
A V E R A G E (Xbar)															
	89.38789	65.762047	53.848437	53.780870457	42.0198274448	38.0281885839	32.581852	53.0468854	51.59670423	80.872738	45.80725161	49.89134065	68.25859	71.99883	68.80085

Note : JC stands for Journey Cycle and is approximately 1 calendar month.

The model developed is thus strategically sufficient since it is linked to the overall objective of profitability and hence overcomes one of the weaknesses of most Productivity measurement models. The computed Productivity values are then converted to unitless scores through Utility function curves (see Table 2.0).

Productivity (in unitless scores) is subsequently regressed with all tangible variables (in unitless scores). At this stage the tangible variables are not separated into "Effectiveness" and "Efficiency" criteria but are considered as individual variables in the regression equation. The multiple regression is done using Principal Component Analysis (PCA) in order to remove the effects of multi collinearity between the tangible variables. This overcomes a major weakness noticed in past models where independence was assumed and no attempt was made to justify it mathematically. Fisher's test is utilised to test significance for overall regression and student's "t" test is utilised in the backward elimination process. The overall regression is significant at 5% level and the coefficient of determination is 0.6967. The number of tangible variables is effectively reduced by considering only those that are significant. The results of the computation is reflected in Table 3.0 with only 8 tangible variables that are found to be significant. All tangible variables belonging to a qualitative criteria (ie. "Effectiveness" or Efficiency" as seen in Fig 1.0) are grouped and the weights (ie., coefficients) subsequently computed (weights refer to the extent of contribution of the tangible variables to the Qualitative criteria and the contribution of the Qualitative criteria to Productivity). The mathematical derivation of weights represents a significant deviation from other standard research work which utilises human judgement and interaction. The derived Productivity measurement model is :

$$Y = 31.376 + 1.03(0.209 \times 7 - 0.02 \times 11 + 0.634 \times 13 + 0.177 \times 15) + 0.387(0.166 \times 3 + 0.267 \times 4 + 0.160 \times 5 + 0.408 \times 6) \quad (1)$$

where,

X7 = Asset utilisation	}	"Effectiveness" criteria
X11 = Inventory level of Finished goods		
X13 = % Production target fulfilled		
X15 = Asset turnover ratio		
X3 = % material variance	}	"Efficiency" criteria
X4 = % labour variance		
X5 = % No production hrs due to no labour, no material and services/available hrs		
X6 = % No production hrs due to machine breakdowns / available hrs		

Reducing the above equation further by grouping the tangible variables to its respective Qualitative criteria,

$$Y = 31.376 + 1.03 \times \text{"Effectiveness" score} + 0.387 \times \text{"Efficiency" score}$$

The measurement model derived proves it's usefulness in the following ways :

- "Effectiveness" contribution towards Productivity is approximately 2.7 times that of "Efficiency". The Organisation should therefore focus towards "Effectiveness" and fulfil demands and requirements of external sub systems.
- Identification of the "top five" variables (ie. the extent of contribution of the tangible variables towards Productivity). This would help the Organisation in it's productivity improvement programmes by focusing the programmes to optimises the "top five" variables. The "top five" variables in descending order are :
 - % Production target fulfilled
 - Asset utilisation
 - Asset turnover ratio
 - % no production hrs due to m/c breakdowns/Available hrs
 - % labour variance

The above 5 variables are an indication of the extent of unutilised assets. The Organisation's aim therefore is to optimise the utilisation of assets and postpone expensive investment.
- The model could be utilised as a measurement and analysis tool. This would bring in a scheme of Productivity audit into the Organisation.

Fig 3.0 depicts the movement of Productivity, "Effectiveness" and "Efficiency" for the periods Jan'94 to July'94. Nineteen periods (or months) were considered for Fig 3.0 in order to clearly illustrate the movements over time. The tangible variables (see equation (1)) for each control period (ie. month) for 1994 are obtained. These tangible variables are then converted to unitless scores using the derived utility function curves and substituted into equation (1). Table 4.0 reflects the changes in Productivity, "Effectiveness", "Efficiency" and individual tangible variables for the periods Jan'94 to July'94 and is used for analysis and measurement purposes for the year 1994 (data shown upto July 1994).

3.0 Optimisation Model

Another major contribution of this research exercise is it's use as an optimisation or deterministic tool. Technological criteria such as effective assets utilisation and machine failure rates are determined for the future by optimising the measurement model.

Table 3 - Results of PCA

Regression Output(PCA):

Constant	48.079671
Std Err of Y Est	11.867613
R Squared	0.6967294
No. of Observations	35
Degrees of Freedom	31

PCA Coefficient(s)	0.331622	0.2044851	0.2761694
Std Err of Coef.	0.043484	0.0747324	0.10275682
Significant @ 5%(t test)	Yes	Yes	Yes

X Coefficients								
X Coefficients	0.064165	0.1032777	0.06181665	0.1580555343	0.21531714829	-0.0211108582	0.6532434	0.18226356
Constant	-31.3761							

Fisher's test

	Sum of squares	
Regression	10030.494	3343.4978
Residual	4366.0477	140.84025

H0= Data is not significant H1= Data is significant

F statistic =	23.7396
F ratio is =	3.8710
@5%	

Therefore the Regression is significant at 5% level of significance

Table 4 - Changes in Criteria Tangible Variables

Control Period	"Effectiveness"				"Efficiency"				"Effectiveness" score	"Efficiency" score	Productivity score
	Tangible var.(X7)	Tangible var.(X11)	Tangible var.(X13)	Tangible var.(X15)	Tangible var.(X3)	Tangible var.(X4)	Tangible var.(X5)	Tangible var.(X6)			
Period 1	42.94	0.00	85.50	68.30	63.10	66.85	20.81	48.84	75.31	51.53	66.13
Period 2	27.79	9.14	73.42	17.89	44.05	59.53	16.82	33.19	55.37	39.40	40.90
Period 3	36.39	13.46	45.34	17.60	55.69	30.87	12.17	27.03	39.21	30.43	20.79
Period 4	29.64	23.35	66.22	77.12	63.74	73.87	7.57	32.37	61.38	44.68	49.13
Period 5	33.96	18.68	74.35	70.13	39.11	79.24	38.82	32.75	66.30	47.17	55.16
Period 6	45.34	44.26	88.65	19.23	43.40	72.42	15.02	39.71	68.21	45.10	56.33
Period 7	45.50	71.52	89.14	24.17	46.64	70.92	23.81	32.01	68.88	43.50	56.40

These technological criteria are the decision variables in the optimisation model. The mathematical formulation of the optimisation model is built up using the stock accumulation formula depicted in Fig 4.0. The stock accumulation formula is a Fourier series when the axes are rotated and requires the average waiting time in the system due to machine breakdowns () and the assumption that the machine failure is Exponential. Due to complexities of the manufacturing system, the average waiting time in the system is computed using simulation [8] [9] [10] and the Exponential distribution of machine failure is proved using Bartlett's test [11]. The formula is thus used to represent the objective function variables and the constraints mathematically in terms of the machine failure rate and effective assets utilisation.

The Objective function is thus - Maximise Yearly Productivity score (Sunned on a monthly basis)

The relevant constraints - (on a monthly basis) Sales, Inventory level of finished products, failure rates, Effective assets utilisation and the working capital.

In the case study, 10 production lines were considered. Lines 1 - 2 produced one brand of Laundry Soaps whilst Lines 3 - 10 produced a more popular brand. The above optimisation model was developed for the case study and is non linear in nature. The objective function is the yearly productivity score (sunned on a monthly basis) whilst the constraints are considered on a monthly basis. (This explains why the objective function values are in excess of 100 as indicated in Table 5.0). The optimisation model is solved using Sequential Linear Programming (SLP) and the technical criteria derived for the planning periods 1994 and 1995 are depicted in Table 5.0.

4.0 Conclusions

This research exercise was done in two stages. Stage 1 consisted of developing a Productivity measurement model incorporating the strengths of Quantitative and Qualitative models. In stage 2, the measurement model is utilised as an Optimisation tool to derive technological criteria.

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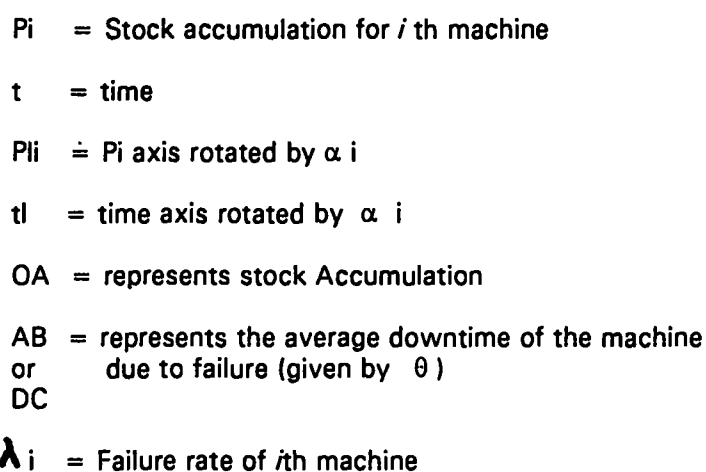


Figure 4 - Cumulative Production for the ith series with rotated axes

Table 5 - Results of the Sequential Linear Programming

	1993 (base)	1994	1995
Objective function value (with c = -230.579*)	632.506	802.545	980.75
Failure rate (per hour)			
Line 1	0.22700	0.22076	0.19900
Line 2	0.18486	0.17978	0.16180
Line 3	0.11248	0.08690	0.07995
Line 4	0.18991	0.13745	0.12645
Line 5	0.24993	0.20624	0.18974
Line 6	0.29140	0.22733	0.20910
Line 7	0.14745	0.10360	0.09532
Line 8	0.17640	0.13401	0.12330
Line 9	0.16556	0.10266	0.09444
Line 10	0.26530	0.19012	0.17490
Effective utilisation for lines 1-2 (%)			
JC 1	37.412%	9.021%	29.298%
JC 2	38.015%	31.762%	35.409%
JC 3	38.861%	32.781%	34.805%
JC 4	25.289%	28.002%	26.359%
JC 5	34.205%	24.565%	24.677%
JC 6	40.640%	26.467%	26.696%
JC 7	41.545%	35.431%	37.618%
JC 8	37.047%	32.096%	34.078%
JC 9	35.485%	33.923%	33.207%
JC 10	40.383%	33.065%	32.858%
JC 11	41.186%	33.537%	31.393%
JC 12	33.380%	27.790%	31.895%
Effective utilisation for lines 3-10 (%)			
JC 1	37.412%	27.592%	35.901%
JC 2	38.015%	40.006%	46.028%
JC 3	38.861%	42.374%	49.875%
JC 4	25.289%	27.866%	31.694%
JC 5	34.205%	42.298%	46.664%
JC 6	40.640%	43.899%	48.343%
JC 7	41.545%	42.383%	46.397%
JC 8	37.047%	40.921%	45.129%
JC 9	35.485%	40.328%	44.490%
JC 10	40.383%	40.971%	44.461%
JC 11	41.186%	43.716%	46.210%
JC 12	33.380%	42.712%	46.008%

* C = The value taken from calculation using 1993 figures

Note : JC stands for Journey Cycle and is approximately 1 calendar month