24

EVALUATION OF THE RELIABILITY OF POWER SYSTEMS USING THE IMPROVED D C LOAD FLOW TECHNIQUE

by

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Abstract

The paper presents a method for the evaluation of reliability of power systems with the Improved D C Load Flow technique providing the base case solution (point of linearisation) for the stochastic load flow calculations. The linearisation is then carried out using a partially recoupled Jacobian Matrix.

The proposed method has much potential for contingency studies on account of its fast execution time and reduced storage. The reliability indices obtained using this method for the IEEE 14 Bus Test System compares very favourably with those using the Fast Decoupled Load Flow technique, as the point of linearisation. The indices are also evaluated for a 46 bus configuration of the Sri Lanka Power System.

List of Principal Symbols

- B_{ik} - susceptance of line ik
- [B′] - line susceptance matrix corresponding to [H]
- [B″] - line susceptance matrix corresponding to [L]
- CL - critical load
- d_o - mean duration of state N_o
- d_j - duration of state N_j
- D - number of days considered
- D_t - duration of system load in excess of critical load
- e - duration of daily peak state
- F( ) - frequency of occurrence of state ( )
- [H] - top diagonal sub-matrix of Jacobian
- j - contingency states
- [J] - recoupling sub-matrix of [J]
- K_{mn} - capacity of line mn
- L_i - peak load state, day i
- L_o - low load state of system
- [L] - bottom diagonal sub-matrix of Jacobian
- n_i - number of occurrences of L_i
- N_o - network normal state
- N_j - network contingency state j
- OL_{mn} - overload state, line mn
- Pr( ) - Probability of state ( )
- P_i - active power, state L_i
- ΔP - active power mismatch
- P_{ik} - active power flow of line ik
- Q_i - reactive power, state L_i
- ΔQ - reactive power mismatch
- Q_{ik} - reactive power flow, line ik
- Q_{ik}^m - modified Q_{ik}
- R_k^{mn} - sensitivity factor, line mn at bus k
- r_{ik} - resistance of line ik
- S_i - complex power, state L_i
- S_{mn} - magnitude of flow, line mn
- UE_j - unserved energy, state j
- V_i, V_k - voltages at ends i and k

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ΔV - voltage magnitude correction
x_{ik} - reactance of line ik
y_{ik}^{sh} - shunt admittance, ik
α, β - deterministic vectors which distribute the load
θ_{ik} - voltage angle across line ik
θ_{ik}' - modified θ_{ik}
$\Delta\theta$ - voltage angle correction
ρ_i - random variable defining the system load state L
μ_i, σ_i - mean value & std. deviation of ρ_i
λ_j - failure rate, state N_j
μ_j - repair rate, state N_j

1.0 Introduction

The majority of methods[1,2] available for the evaluation of reliability of power systems use repetitive load flow studies for each contingency. This results in excessive computation time. In fact in most of these methods, the bulk of the computation time is utilised in performing the load flow analysis[3]. in order to cut down the computation time required, most methods use approximate load flow techniques because they are fast. The D C Load Flow technique, although very fast, has not been used much in the past because it provided estimates of line active power flows only. It provided no estimate of bus voltages, essential for the calculation of reliability indices of individual system buses[3].

Karunaratne et al[4,5] have shown that the d.c. network analyser may be used to determine these quantities. Lucas et al[6] have shown based on the postgraduate work of Malembeka[7], that the Improved D C Load Flow (IDCLF) technique is capable of providing both estimates of line power flows and the bus voltages on the digital computer.

This paper presents a method in which this Improved D C Load Flow technique is used in the evaluation of reliability of two power systems, namely the IEEE 14 Bus Test Power System[8] and a 46 Bus configuration of the Sri Lanka Power System[9].

It further describes how the output of the Improved D C Load Flow technique is used to provide the point of linearisation (base case solution) for stochastic load flow calculations. The reliability indices obtained are compared with those obtained by Billinton et al[3] and Induruwa[1] who have used the output of the FDLF technique to provide the base case solution.

2.0 The Deterministic Load Flow Analysis

In this method, a deterministic load flow study is performed for the base case for each contingency. The output of this is used to provide a point of linearisation for the stochastic load flow calculations. For this purpose, Induruwa[1] and Billinton[3] have used the output of the FDLF technique. In this paper the Improved D C Load Flow technique is used instead to provide the base case.

The basic equations of the technique[6] are given in equation 2.1 and 2.2.

$$[P] = [H]\,[\theta'] \qquad \text{----}(2.1)$$
$$[Q^m] = [L']\,[V] \qquad \text{----}(2.2)$$

where

$$\theta_{ik}' = V_i V_k\,(\theta_{ik} + (r_{ik}/x_{ik})*(V_i - V_k)) \qquad \text{----}(2.3)$$

and

$$Q_i^{\,m} = Q_i^{\,sp} + \sum_{k\in i}(-P_{ik}\theta_{ik}'/2 + (r_{ik}/x_{ik})P_{ik} + y_{ik}^{sh}/2) \qquad \text{----}(2.4)$$

3.0 The Stochastic Load Flow Calculations

In orthodox methods of stochastic load flow calculations, the critical load for a contingency state is obtained by performing load flow studies repetitively using different load levels within a predicted range. In some cases as many as 10 load flow studies are required to establish the critical load for a particular contingency state[2].

In the method presented in this paper, and in the methods presented by Induruwa[1] and Tom Hayes[2], the problem of repetitive load flow studies is eliminated by the use of a stochastic load flow (SLF) method.

For the majority of contingencies, the critical load is obtained using one load flow study. Only in cases where a contingency produces a serious overload, requiring a reduction of the system load, is an extra load flow study required[1]. The system load and the system network are therefore modelled as stochastic processes.

3.1 System Load Model

The system load model is presented as a vector, first order Markov process to include the uncertainities of load forecasting and the sequence of daily peak loads. The daily load cycle is approximated by the daily peak load states L_i, each of duration e (<1) days alternating with periods of low load L_0 of duration $(1-e)$ days, as shown in Figure 3.1.

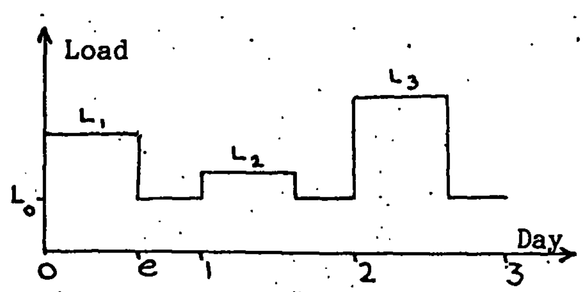

Figure 3.1 Daily Load Cycle

The factors which influence the daily peak load states are factors such as weather, day of week and holidays. Although these cannot be predicted exactly, average values of variations can be obtained[1]. Each loadstate L_i, is defined by a random vector S_i given by

$$S_i = P_i + j\,Q_i = (\alpha_i + j\,\beta_i)\,\rho_i \quad \text{----}(3.1)$$

In general, it may be assumed that the distribution factors α_i and β_i are equal for all load states, so that the individual bus loads are modelled as being perfectly correlated.

$$\text{i.e.} \quad \alpha_i = \alpha\,, \ \beta_i = \beta \quad \text{----}(3.2)$$

The load is therefore now a deterministic function of the random variable ρ_i, with mean μ_i and variance σ_i^2, representing the uncertainty in the load. Since the process is assumed to be ergodic, the probability of existence of a load state is equal to the average fraction of time spent in that state. The sequence of peak loads is random however, and the average number of occurrences n_i of each peak load state during the period D is assumed to be known. Thus

$$D = \sum_{i=1}^{I} n_i \quad \text{----}(3.3)$$

The probability that the load state L_i exists is

$$Pr(L_i) = n_i \cdot e/D, \ i \neq 0 \quad \text{----}(3.4)$$
$$\text{and} \quad Pr(L_o) = 1 - e \quad \text{----}(3.5)$$

The expected frequency of occurrence of load state L_i is

$$F(L_i) = n_i/D, \quad i \neq 0 \quad \text{----}(3.6)$$
$$\text{and} \quad F(L_o) = 1 \quad \text{----}(3.7)$$

3.2 Transmission Network Model

The network is modelled as a load independent first order Markov process. This assumes statistically independent, stationary, exponential distributions of network contingencies and repair times. The network state N_o, figure 3.2, is the state with all the components operating normally. The network moves from state N_o to contingency states N_j (for j - 1, 2, ... J) with failure rates j and returns from contingency state to normal state with repair rate λ_j. (The general assumption is made that the failure rate λ_j is a function of operating voltage and length, and that the repair rate μ_i is a function of operating voltage only).

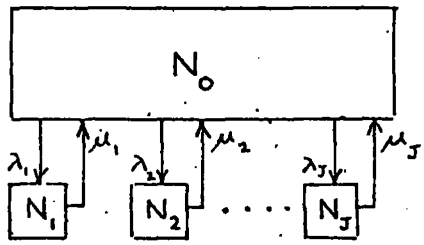

Figure 3.2 Network Transition Diagram

The mean duration of contingency is

$$d_j = 1/\mu_j \qquad \text{----(3.8)}$$

and the probability of state N_j existing at some time in the future is

$$Pr(N_j) = \lambda_j/(\lambda_j + \mu_j) \qquad \text{----(3.9)}$$

The expected frequency of occurrence of state N_j is

$$F(N_j) = \mu_j \, Pr(N_j), \; j \neq 0 \qquad \text{----(3.10)}$$

and of the state N_o is

$$F(N_o) = \sum_{j=1}^{J} F(N_j) \qquad \text{----(3.11)}$$

The probability of existence of state N_o is

$$Pr(N_o) = d_o . F(N_o) \qquad \text{----(3.12)}$$

4.0 Stochastic Load Flow using the System Load Model

For a given set of busbar and network conditions, the deterministic load flow analysis gives a discrete set of bus voltages (V_o, θ_o) and power flows (P_o, Q_o) as a base case solution. To obtain probabilistic values from these results, the parameters of the load model developed must be obtained from a stochastic load flow technique.

The load flow equations 2.1 and 2.2 do not provide sufficient accuracy for linearisation. Thus partially recoupled equations are used. Here, the reactive power flow is recoupled to the voltage angles so that the FDLF equations become

$$\begin{bmatrix} \Delta P/V \\ \Delta Q/V \end{bmatrix} = \begin{bmatrix} B' & 0 \\ J & B'' \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \qquad \text{----(4.1)}$$

where the elements of the added submatrix [J] are given by

$$J_{ik} = - V_k B_{ik} \sin \theta_{ik}$$

Solution of equation 4.1 gives

$$[\Delta\theta] = [B']^{-1}[\;P/V] \qquad \text{----(4.2)}$$
$$[\Delta V] = -[B'']^{-1}[J][B']^{-1}[\Delta P/V] + [B'']^{-1}[\Delta Q/V] \qquad \text{----(4.3)}$$

Taking the base case as the point of linearisation gives the stochastic variables as $\theta + \Delta\theta$, $V + \Delta V$, $\rho_o + \Delta\rho$, $P_o + \Delta P$ and $Q_o + \Delta Q$. We now need to relate the other variations with the variation in the random variable $\Delta\rho$.

From equation 3.1, it follows that

$$[\Delta P] = [\alpha]\Delta\rho \text{ and } [\Delta Q] = [\beta]\Delta\rho \quad \text{--(4.4)}$$

Subtitution in equation 4.2 gives

$$[\Delta\theta] = [B']^{-1}[\alpha/V]\Delta\rho$$
or
$$= [Z_1]\Delta\rho \quad \text{----(4.5)}$$

and substitution in equation 4.3 gives

$$[\Delta V] = \{[B'']^{-1}[J][Z_1]+[B'']^{-1}[\beta/V]\}\Delta\rho$$
or
$$= [Z_2]\Delta\rho \quad \text{----(4.6)}$$

4.1 Probability of Bus Voltages violating its limits

Since the bus voltage generally drops when the system load is increased, the probability of the bus voltage violating its lower limit (V_{KL} at bus K) involves the calculation of the probability that the system load represented by the random variable ρ exceeds a critical value ρ_{c1}. Thus

$$\Delta\rho_k = (V_{kL} - V_{ok})/Z_{2k} \quad \text{----(4.7)}$$

from which

$$\rho_{kc} = \rho_{c1} + (V_{kL} - V_{ok})/Z_{2k} \quad \text{----(4.8)}$$

This gives the probability of bus under-voltage as

$$Pr(V_k < V_{kL}) = Pr(\rho > \rho_{kc1})$$

Similarly, the probability of bus voltage violating its upper limit (V_{kU} at bus k) is given as

$$Pr(V_k > V_{kU}) = Pr(\rho < \rho_{kc2})$$

4.2 Probability of Line Overload

The active power flow P_{mn} and the reactive power flow Q_{mn} in line mn are used in the calculation of the line overloads. They are linearised around the base case solution P_{mno} and Q_{mno}.

$$P_{mn} = P_{mno} + \Delta P_{mn}, \quad Q_{mn} = Q_{mno} + \Delta Q_{mn}$$

The increments P and Q are related to by P = E , and Q = F (See Appendix A1).

The power flow S_{mn} in line mn is given by $S_{mn}^2 = P_{mn}^2 + Q_{mn}^2$, which may be expressed around the base case solution as

$$S_{mn}^2 = (E_{mn}^2+F_{mn}^2)\Delta\rho^2 + 2(P_{mno}E_{mn} + Q_{mno}F_{mn})\Delta\rho + P_{mno}^2 + Q_{mno}^2 \quad \text{----(4.9)}$$

The probability of line overload thus involves calculating the system load which causes the line flow to exceed the maximum capacity K_{mn}. Thus for line mn overloading, $S_{mn}^2 > K_{mn}^2$ so that

$$(E_{mn}^2+F_{mn}^2)\Delta\rho^2+2(P_{mno}E_{mn}+Q_{mno}F_{mn})\Delta\rho + P_{mno}^2 + Q_{mno}^2 - K_{mn}^2 > 0 \quad \text{----(4.10)}$$

If ρ_L and ρ_U with $\rho_L < \rho_U$, are the values of which satisfy the limiting case of the inequality, then the probability of line overload is given by

$$Pr(OL_{mn}) = Pr(\rho < \rho_L) + Pr(\rho > \rho_U).$$

The probabilities which have been obtained so far refer to the conditions of network elements resulting from a failure of a particular network element. However, they do not reflect the influence of these failures on the probability of failure of customer load points.

5.0 Calculation of Probabilities of Failure of Customer Load Points

The service interruption at a customer load point is defined as the condition that a demand exists which cannot be satisfied due to overloaded lines and/or unacceptable bus voltages [1]. Thus the probability of failure of customer load points involves the use of probabilities of line overloads and of bus voltages violating their limits. However due to the interconnected nature of the modern high voltage transmission networks, only the failure of a set of transmission lines directly contribute to the failure at a consumer load point. Therefore the evaluation of bus failure probabilities requires the identification of those lines which carry a significant part of the load supplied to a bus in question. This can be done by evaluating the sensitivity of power flows in lines to the power demand at a bus[1, 10].

This sensitivity factor is given by

$$R_k^{mn} = -B_{mn}\,(b''_{mk}{}^{-1} - b''_{nk}{}^{-1}) \quad \text{--(5.1)}$$

(See Appendix A2 for details)

The relative magnitude of the sensitivity factors indicate the amount of reactive power carried by the line to a particular load point. If R_k^{mn} is positive, then that line is carrying reactive power away from the bus.

With the probabilities evaluated in section 4.1 and section 4.2, and the sensitivity factors, the probability of failure is obtained[1] as follows.

The probability of failure at bus k, Pr_k is defined as the ratio of the expected total duration of supply interruption at bus k, to total time period. Thus

$$Pr_k = \sum_{i=0}^{I} \sum_{j=0}^{J} Pr(L_i)Pr(N_j)\ Pr_{ijk} \quad \text{----(5.2)}$$

Pr_{ijk} is the conditional probability that the load at bus k will not be satisfied, conditioned on the load state i and the network state j. Since satisfying the demand at bus k requires acceptable bus voltages and availability of all necessary transmission network elements, the value of Pr_{ijk} is calculated as the maximum of the probabilities of unacceptable bus voltage and overloads of individual transmission elements supplying the demand at bus k. (This is analogous to equating the strength of a chain to the strength of its weakest link). Thus

$$Pr_{ijk} = \max\{Pr(V_k<V_{kL}),\ Pr(V_k>V_{kU}),\ Pr(OL_{mnpq})_D\} \quad \text{----(5.3)}$$

where $Pr(OL_{mnpq})$ is the probability of overload of line mn due to failure of line pq, and suffix D is the set of lines which have the largest line flow to bus power sentivities.

Similarly, the expected frequency of failure at bus k is

$$F_k = \sum_{i=0}^{I} \sum_{j=0}^{J} F_{ij}\ Pr_{ijk} \quad \text{----(5.4)}$$

where F_{ij} is the expected frequency of occurrence of the combined event of load state i and the network state j, defined as follows.

$$F_{ij} = Pr(L_i)\ Pr(N_j)\ [1/e + 1/d_j]$$

$$F_{oj} = Pr(L_o)\ Pr(N_j)\ [1/(1-e) + 1/d_j]$$

$$F_{io} = Pr(L_i)\ Pr(N_o)\ [1/e + 1/d_o]$$

$$F_{oo} = Pr(L_o)\ Pr(N_o)\ [1/(1-e) + 1/d_o]$$

5.2 Overall System Performance Indices

In addition to evaluating the performance indices of consumer load points, based on the existence of particular load and network states, it is necessary to evaluate the overall system reliability performance indices. These are the overall service failure probability and the expected unserved energy. These two, among other indices, give a reasonable estimate of adequacy of the composite generation and transmission system.

5.2.1 Calculation of Service Failure Probability

A service failure state is defined

as a condition which causes service interruption at a consumer load point due to overloaded lines and/or unacceptable bus voltages. These interruptions generally result in either complete or partial loss of load at one or more consumer load points, due to the load reductions necessary to alleviate line overloads or inadequate voltages. This implies that for each contingency state there exists a critical load level L_c which, if exceeded, results in a service interruption.

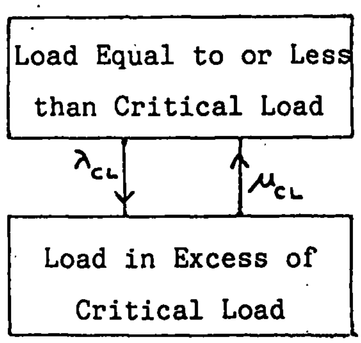

Figure 5.1 Two-state Load Model

Therefore in the evaluation of service probability, the load can be presented as a two state model, as shown in figure 5.1. Billinton et al[3] have developed such a model using historical load data. The transition rates for this model are given by equation 5.5.

$$\mu_{CL} = (n/Dt).T$$
$$\lambda_{CL} = n/(1 - Dt/T) \qquad \text{----(5.5)}$$

where T is the time period, and n is the number of times the load exceeds CL during period T.

Using these transition rates, the probability of the load exceeding the critical load level CL could be obtained as

$$\begin{aligned} Pr(Lc) &= \lambda_{CL}/(\lambda_{CL} + \mu_{CL}) \\ &= Dt/T \qquad \text{----(5.6)} \end{aligned}$$

The service failure probability due to the combined effect of network contingency state j and the corresponding critical load level L_{cj} can be expressed as

$$Pr_j^{sf} = Pr(N_j).\ Pr(Lc_j) \qquad \text{----(5.7)}$$

Therefore, the overall system failure probability P_{rT}^{sf} can be written as

$$Pr_T^{sf} = \sum_j Pr_j^{sf} \qquad \text{----(5.8)}$$

and the system failure duration

$$T^{sf} = Pr_T^{sf}.T \quad \text{hrs/year} \qquad \text{----(5.9)}$$

5.2.2 Calculation of Unserved Energy

The unserved energy UE, due to the service interruption caused by network contingency state j can be calculated from the load model as

$$UE_j = (PD - Lc_j)\ D \quad \text{MWh} \qquad \text{----(5.10)}$$

Thus the total unserved energy is

$$UE_j = \sum_{Kj} (PD - Lc_j)\ D \quad \text{MWh} \qquad \text{----(5.11)}$$

The expected unserved energy per year is given by

$$E(UE_j) = (1/Y_{Kj}) \sum_{Kj} (PD-Lc_j)/8760 \ \text{MWh/yr}$$

where PD is the maximum power demand in MW, K is a contingency state whose critical load level L is less than the maximum power demand PD, and Y is the number of contingency states whose critical load is less than the maximum power demand PD. In calculating these indices, the critical load L_{cj} must first be ascertained.

In the method presented by Marks[9], the critical load level is obtained by performing a statistical load flow calculation for the system peak load. If a line is overloaded, the system load is reduced to the capacity of the line. This involves a number of load flow calculations which is time wasting.

In the method presented by Induruwa[1], and also used in this work, the critical load level of the majority of contingencies could be obtained by repeating the load flow calculations only once. The procedure involved is presented in Appendix A3.

6.0 Systems considered in the Study

The method presented in this paper was tested by performing reliability studies for two power systems, namely the IEEE 14 Bus Test Power System[8] and a 46 bus configuration of the Power System in Sri Lanka [9]. These are shown in Figure 6.1 and Figure 6.2 respectively.

The network elements considered for the two systems are given in Table 6.1 and Table 6.2 respectively.

7.0 Results

Tables 7.1 and 7.2 show the reliability indices for the IEEE 14 Bus Test Power System. Table 7.1 shows the comparison of the probabilities of bus failure, while table 7.2 shows a comparison of the frequencies of bus failure obtained by the different methods [1,2]. The results of the study indicate that the reliability indices for the IEEE 14 bus test system calculated by the three methods are numerically different. The difference between the indices obtained by the different methods can be attributed mainly to the fact that the system load is modelled differently, and that some of the data used in the studies could be different. In the present study, although the system load and network are modelled the same as by Induruwa [1] with the outage data

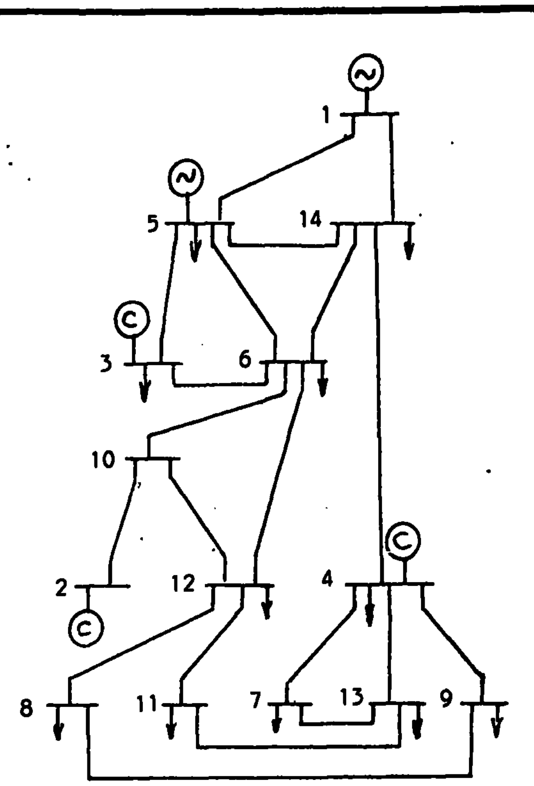

Figure 6.1 IEEE 14 Bus Test System

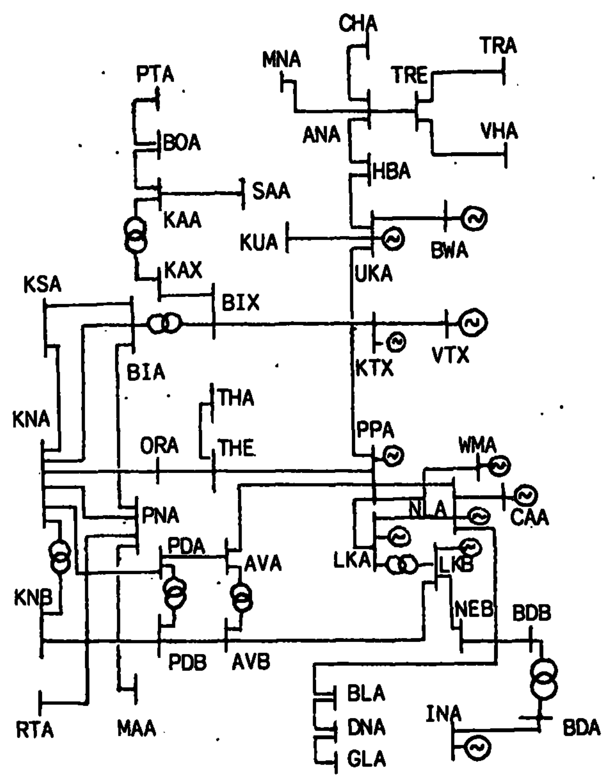

Figure 6.2 46 Bus Sri Lanka System

TABLE 6.1 Outage Data For IEEE 14 Bus Test Power System

Line	SBus	EBus	λ Occ/Yr	d hrs	Pr(j) x10⁻³
1	1	5	1.06	10	1.21
2	1	14	2.00	10	2.28
3	6	10	0.00	10	0.00
4	3	5	1.78	10	2.03
5	3	6	1.53	10	1.75
6	4	7	1.47	10	1.68
7	4	9	1.14	10	1.30
8	4	13	0.75	10	0.85
9	4	14	0.04	336	1.53
10	5	6	1.58	10	1.80
11	5	14	1.56	10	1.78
12	10	2	0.00	10	0.00
13	6	12	0.04	336	1.53
14	6	14	0.38	10	0.43
15	7	13	1.15	10	1.31
16	8	9	1.10	10	1.26
17	8	12	0.49	10	0.56
18	10	12	0.00	10	0.00
19	11	12	1.55	10	1.77
20	11	13	2.00	10	2.28

λ - failure rate
d - repair duration
Pr(j) - probability of failure

TABLE 6.2 Outage Data For 46 Bus Sri Lanka Power System

	Line	λ Occ/yr	d hrs	Pr(j) x10⁻³
1	KAA-SAA	0.19	20	0.43
2	KAA-SAA	0.17	20	0.39
3	BIA-KSA	0.14	20	0.32
4	KSA-KNA	0.02	20	0.04
5	PNA-RTA	0.05	20	0.11
6	PPA-THE	0.23	20	0.52
7	THE-ORA	0.29	20	0.66
8	PPA-AVA	0.27	20	0.62
9	AVA-PDA	0.88	20	2.01
10	PPA-UKA	0.49	20	1.12
11	BWA-UKA	0.06	60	0.41
12	UKA-HBA	0.15	60	1.01
13	HBA-ANA	0.09	60	0.62
14	ANA-TRE	0.13	60	0.89
15	TRE-TRA	0.07	60	0.48
16	TRE-VAA	0.14	60	0.96
17	ANA-MNA	0.15	60	1.03
18	ANA-CHA	0.36	60	2.49
19	BIA-KNA	0.14	20	0.32
20	LKA-PPA	0.07	36	0.29
21	NLA-PPA	0.07	36	0.29
22	NLA-LKA	0.01	36	0.04
23	NLA-BLA	0.41	36	1.68
24	BLA-EMA	0.40	36	1.64
25	BLA-DNA	0.41	36	1.68
26	DNA-GLA	0.51	36	2.09
27	BDA-INA	0.74	36	3.04
28	NLA-CAA	0.09	36	0.37
29	BOA-PTA	0.68	36	2.79
30	UKA-KUA	0.07	60	0.48
31	KNA-PNA	0.11	20	0.25
32	BIA-PNA	0.14	20	0.32
33	PNA-MAA	0.39	20	0.89
34	THE-THA	0.19	20	0.43
35	ORA-KNA	0.11	20	0.25
36	PDA-KNA	0.21	36	0.86
37	LKB-NEB	0.35	36	1.44
38	NEB-BDB	0.33	36	1.36
39	KNB-PDB	0.27	36	1.11
40	PDB-AVB	0.18	36	0.74
41	AVB-LKB	0.34	36	1.40
42	VTX-KTX	0.30	36	1.23
43	KTX-BIX	0.66	36	2.71
44	BIX-KAX	0.21	36	0.86
45	LKB-LKA	0.02	50	0.11
46	KNB-KNA	0.02	50	0.11
47	BIA-BIX	0.02	50	0.11
48	KKA-KAX	0.02	50	0.11
49	BDB-BDA	0.02	50	0.11
50	PDB-PDA	0.02	50	0.11
51	AVB-AVA	0.02	50	0.01

λ - failure rate
d - repair duration
Pr(j) - probability of failure

TABLE 7.1 Probability of Bus Failure for IEEE 14 Bus Test System

Bus No.	Probability of Bus Failure x 10^{-3} Ref 1	Ref 2	IDCLF
1	-	-	-
2	-	-	-
3	-	-	-
4	-	-	-
5	-	-	-
6	1.16	2.62	6.33
7	4.61	2.45	2.86
8	4.19	7.52	6.58
9	4.63	3.28	6.58
10	-	-	-
11	1.47	1.38	4.51
12	4.50	6.99	5.61
13	4.43	2.99	2.77
14	1.90	1.46	6.15

TABLE 7.2 Frequency of Bus Failure For IEEE 14 Bus Test System

Bus No	Frequency of Bus Failure x 10^{-3} Ref 1	Ref 2	IDCLF
1	-	-	-
2	-	-	-
3	-	-	-
4	-	-	-
5	-	-	-
6	1.45	2.43	3.19
7	5.72	1.44	1.72
8	5.24	5.33	3.03
9	5.79	2.22	2.69
10	-	-	-
11	1.83	10.34	2.35
12	5.64	4.83	2.87
13	5.53	1.95	1.68
14	2.37	1.36	2.99

TABLE 7.3 Probability of Bus Failure & Frequency of Failure for 46 Bus Sri Lanka System

Bus	Notation	Pr(j) x10^{-3}	F(j) per Yr
12	KNB	0.08	0.04
13	KNA	0.08	0.04
14	KSA	0.13	0.33
15	SSA	0.45	0.61
16	KAX	0.28	0.53
17	KAA	0.28	0.52
18	BOA	0.61	0.81
19	PTA	0.28	0.54
20	PNA	0.62	0.82
21	RTA	0.74	1.63
22	AVB	0.01	0.25
23	AVA	0.01	0.25
24	PDB	0.19	0.81
25	PDA	0.14	0.81
26	ORA	0.01	0.49
27	THE	0.67	1.24
28	THA	0.99	2.74
29	MAA	0.01	0.00
30	KUA	0.39	0.13
31	HBA	5.08	6.31
32	ANA	5.22	6.65
33	MNA	0.23	0.04
34	TRA	7.03	8.01
35	VAA	0.21	0.04
36	CHA	3.55	8.01
37	BLA	0.74	0.04
38	EMA	0.06	4.73
39	DNA	1.24	0.55
40	GLA	1.25	0.09
41	NEB	0.00	1.39
42	BDB	0.91	2.21
43	BDA	0.89	1.15
44	BIA	0.35	1.02
45	BIX	0.38	0.43
46	TRE	0.61	0.92

Pr(j) - probability of bus failure
F(j) - frequency of bus failure

also the same, the stochastic load flow calculations are linearised about different points. As a result, the indices calculated are also different.

Both the present study as well as reference 1 gives the same value of 0.136 for the overall probability of service failure. The expected value of unserved energy is 3932 MWh/year for reference 1 and 5641 MWh/year for the present work which are not significantly different.

The reliability indices for the Sri Lanka Power System have been evaluated and are shown in Table 7.3. The overall system failure indices for the Sri Lanka Power System considered are as follows. The Expected unserved Energy is 3275 MWh/year, while the Probability of Service Failure is 0.00073. Although a similar study has been conducted by Induruwa[1], it is on a different configuration and with assumed outage data. Thus comparison is not made with these results. The outage data used in the present study has been obtained[5] for the actual power system in Sri Lanka.

8.0 Conclusions

The method presented in this paper is based on the improved d c load flow (IDCLF) technique to obtain the base case solution. Linearisation around this point has been obtained by partial recoupling of the reactive power with the voltage angle. It has the advantage that the overall speed of the program is increased by about 30%, and the storage requirement is reduced by about 8%.

Comparison of the results for the IEEE 14 Bus Test System considered have shown that the range of indices calculated by the IDCLF method does not differ from the range of indices calculated in the other two studies. Therefore the indices calculated in this study are considered acceptable.

9.0 References

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APPENDIX A1 - Calculation of the Probability of Line Overload

The expressions for active and reactive power flow in line mn are given by

$$P_{mn} = G_{mn}V_m^2 - V_mV_n(G_{mn}\cos\theta_{mn} + B_{mn}\sin\theta_{mn}) \quad \text{----} (A1.1)$$

and

$$Q_{mn} = -B_{mn}V_m^2 - V_mV_n(G_{mn}\sin\theta_{mn} - B_{mn}\cos\theta_{mn}) \quad \text{----}(A1.2)$$

Equations A1.1 and A1.2 are linearised around the base case solution P_{mno} and Q_{mno} as follows.

$$P_{mn} = P_{mno} + \Delta P_{mn} \quad \text{----}(A1.3)$$

$$Q_{mn} = Q_{mno} + \Delta Q_{mn} \quad \text{----}(A1.4)$$

From equations (A1.1) and (A1.2)

$$\Delta P_{mn} = \{2\, G_{mn}\, V_{mo}\, Z_{2m} + V_{mo}\, V_{no}\, (G_{mn} * \sin\theta_{mno} - B_{mn}\cos\theta_{mno})(Z_{1m} - Z_{1n}) - (V_{no}\, Z_{2m} + V_{mo}\, Z_{2n}) * (G_{mn}\cos\theta_{mno} + B_{mn}\sin\theta_{mno})\}\Delta\rho$$

$$= E_{mn}.\Delta\rho \quad \text{----}(A1.5)$$

and

$$\Delta Q_{mn} = \{-2\, B_{mn}\, V_{mn}\, Z_{2m} - V_{mo}\, V_{no}\, (G_{mn} * \cos\theta_{mno} + B_{mn}\sin\theta_{mno})(Z_{1m} - Z_{1n}) - (V_{no}\, Z_{2m} + V_{mo}\, Z_{2n}) * (G_{mn}\sin\theta_{mno} - B_{mn}\cos\theta_{mno})\}$$

$$= F_{mn}.\Delta\rho \quad \text{----}(A1.6)$$

The power flow in line mn is

$$S_{mn}^2 = P_{mn}^2 + Q_{mn}^2$$

$$= (P_{mno} + \Delta P_{mn})^2 + (Q_{mno} + \Delta Q_{mn})^2$$

$$= P_{mn}^2 + Q_{mn}^2 + 2(P_{mno}P_{mn} + Q_{mno}Q_{mn}) + P_{mno}^2 + Q_{mno}^2$$

substituting for ΔP_{mn} and ΔQ_{mn} in terms of $\Delta\rho$ gives

$$S_{mn}^2 = (E_{mn}^2 + F_{mn}^2)\Delta\rho^2 + 2(P_{mno}E_{mn} + Q_{mno}F_{mn})\Delta\rho + P_{mno}^2 + Q_{mno}^2 \quad \text{--}(A1.7)$$

APPENDIX A 2-Calculation of Sensitivity Factors

The sensitivity of the reactive power flow in line mn to the reactive power demand at bus k is determined by the sensitivity factor R_k^{mn}, which is defined by the equation

$$R_k^{mn} = \partial Q_{mn}/\partial Q_k \quad \text{----}(A2.1)$$

But

$$Q_{mn} = \mathrm{Imag}[V_m I_{mn}^*] \quad \text{----}(A2.2)$$

$$= \mathrm{Imag}[V_m^2 - V_mV_n(\cos\theta_{mn} + j\sin\theta_{mn})(G_{mn} - j B_{mn})]$$

$$= -B_{mn}(V_m^2 - V_m V_n \cos\theta_{mn}) - G_{mn}V_mV_n\sin\theta_{mn} \quad \text{----}(A2.3)$$

For small transmission angles $\theta_{mn} \simeq 0$ so that $\cos\theta_{mn} \simeq 1$ and $G_{mn}\sin\theta_{mn} << B_{mn}$ giving

$$Q_{mn} = -B_{mn}(V_m^2 - V_m V_n) \quad \text{----}(A2.4)$$

from which

$$\partial Q_{mn}/\partial Q_k = -B_{mn}[(2V_m - V_n)\partial V_m/\partial Q_k - V_m.\partial V_n/\partial Q_k \quad \text{---}(A2.5)$$

Assuming $V_m = V_n = 1.0$ pu, gives

$$\partial Q_{mn}/\partial Q_k = -B_{mn}[\partial V_m/\partial Q_k - \partial V_n/\partial Q_k] \quad \text{----}(A2.6)$$

From the decoupled power flow model

$$[\partial V] = [B''^{-1}][\partial Q]$$

$$\partial V_m/\partial Q_k = b''_{mk}{}^{-1}$$

and

$$\partial V_n/\partial Q_k = b''_{nk}{}^{-1} \quad \text{----}(A2.7)$$

The sensitivity factor is given by

$$R_k^{mn} = -B_{mn}(b''_{mk}{}^{-1} - b''_{nk}{}^{-1}) \quad \text{---}(A2.8)$$

APPENDIX A3 - Calculation of the Critical Load

If a line is overloaded at the peak load level, the load must be reduced until the line flow is equal to its limit. Since the system load is represented as a random variable[1], the critical value of system load could be obtained[1] by calculating the critical value ρ_c. However, due to linearisation errors, this value of ρ_c will be different from the true value of ρ_c. Hence the load flow calculation must be repeated once more to establish a more accurate load level. If the critical load level ρ_c is not very much different (not more than about 10%) from the peak load, then the critical load obtained will be reasonably accurate and the load flow calculation need not be repeated. However if a particular contingency causes a severe overload which requires a considerable reduction of system load, the procedure will require an additional load flow solution.