A PRACTICAL REALIZATION OF THE VAN DER POL OSCILLATOR

F. V. C. Mendis

Abstract

A source-coupled FET circuit is investigated both theoretically and experimentally. It is shown that the input characteristic closely approximates the cubic $i = -av + bv^3$ so that with the addition of a simple tuned circuit, a Van der Pol oscillator is formed. The performance of this free running oscillator is shown to be in accordance with theory.

1. Introduction

Consider a non-linear element having the cubic characteristic

$$i = -a v + b v^3 \qquad \ldots\ldots(1)$$

where v and i are the voltage and current respectively. The non-linear resistance is therefore given by

$$R(v) = \frac{v}{i} = \frac{1}{bv^2 - a}$$

and it is seen that R(v) is negative in the vicinity of v = o. An oscillator is formed by connecting a RLC parallel circuit across the non-linear element as shown in Fig. 1. It should be noted that this non-linearity is essential for stable operation.

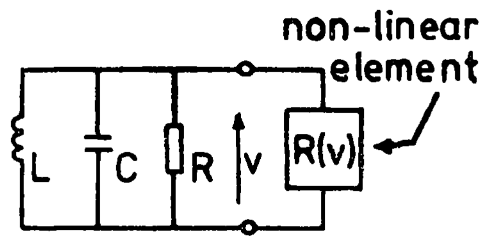


Fig.1

The differential equation describing the circuit may be written in the form

$$\ddot{v} - \alpha\omega_0(1 - \Delta v^2)\dot{v} + \omega_0^2 v = 0 \qquad \ldots(3)$$

$$\text{where} \quad \alpha = \frac{a - 1/R}{\omega_0 C}, \quad \Delta = \frac{3b}{1 - 1/R}$$

$$\text{and} \quad \omega_0 = \frac{1}{\sqrt{(LC)}}$$

The above is the so-called Van der Pol equation[4]. The solution to equation (3) has been shown[2,3] to be

$$v = E\cos\omega_0 t + \frac{\alpha E}{6}(3\sin\omega_0 t - \sin 3\omega_0 t) \qquad \ldots(4)$$

$$\text{where } E = \frac{2}{\sqrt{\Delta}}$$

is the free-running amplitude. This solution is valid for $\alpha << 1$. Also, for stable operation the Q-factor ($Q = R/\omega_0 L$) of the tank circuit, must be high and the inequality

$$\sqrt{(L/C)} \le 1/a < Q\sqrt{(L/C)} \qquad \ldots(5)$$

must hold for $Q \geq 10$.

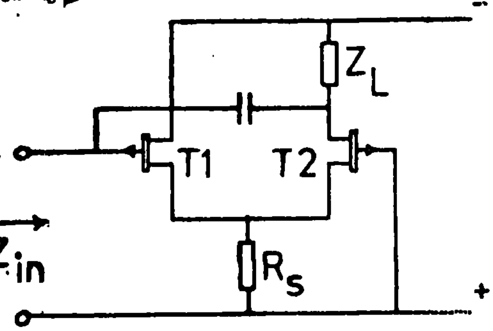
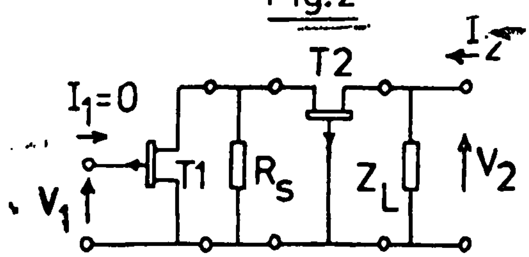


Fig.2



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [A]\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \; ; \quad [A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Fig.3

2. Source-Coupled FET Circuit

It has been remarked[3] that in all the literature relating to the Vander Pol oscillator there appears to be

F. V. C. Mendis (BSc(Eng), MSc, PhD, MIEE, MIE(SL), CENg graduated from the University, Peradeniya, in August, 1966 with first class honours, He joined the Ceylon Broadcasting Corporation in July, 1967 and in September, 1969 was awarded a Commonwealth scholasrhip for postgraduate study in England, He received the MSc, and PhD, degrees from the University of Birmingham, in 1970 and 1973 respectivey, Since 1973, he has been a lecturer in Electrical Engineering in the University, Peradeniya. Dr. Mendis' main interest is in Space Communications.

$$[A] = \begin{bmatrix} \dfrac{g_{m1}+g_{d1}}{g_{m1}} & \dfrac{1}{g_{m1}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R_s} & 1 \end{bmatrix} \begin{bmatrix} \dfrac{g_{d2}}{g_{d2}+g_{m2}} & \dfrac{1}{g_{d2}+g_{m2}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{Z_L} & -1 \end{bmatrix} \quad ..(6)$$

only one reference to a circuit which produces a close approximation to the cubic v-1 characteristic necessary for its realization, this being a twin-triode cathode-coupled circuit.[4] Let us consider an analogous circuit shown in Fig. 2. using field effect transistors (FET). With the feed-back capacitor removed, the circuit can be redrawn for the purposes of analysis, as four 2-port networks in cascade as shown in Fig. 3. The 'A' matrix for this circuit may be written in terms of the A-matrices of the individual networks as given in eqn. (6) above.

Neglecting gd1 and gd2 in comparison with gm1 and gm2 respectively, we have :

$$a_{11} = \frac{1}{Z_L}\left[\frac{1}{g_{m1}} + \frac{1}{g_{m2}} + \frac{1}{g_{m1}\,g_{m2}\,R_s}\right]$$

$$a_{12} = -\left[\frac{1}{g_{m1}} + \frac{1}{g_{m2}} + \frac{1}{g_{m1}\,g_{m2}\,R_s}\right]$$

$$a_{21} = 0$$

$$a_{22} = 0 \quad \ldots(7)$$

This gives $V_2 = G\,V_1 + R_o\,I_2 \quad \ldots(8)$

where $G = \dfrac{1}{a_{11}}$ and $R_o = -\dfrac{a_{12}}{a_{11}}$

With the feed-back applied and assuming that the capacitor C has negligible reactance at the operating frequency, it is simple to show that the input impedance is given by

$$Z_{in} = \frac{R_o}{1-G} = \frac{a_{12}}{1-a_{11}} \quad ..(9)$$

which upon substitution for a_{11} and a_{12} becomes:

$$Y_{in} = \frac{1}{Z_{in}} = \frac{1}{Z_L} - \frac{g_{m1}\,g_{m2}}{g_{m1}+g_{m2}+1/R_s} \quad \ldots(10)$$

which can easily be made negative for normal values of the FET parameters. Thus it is theoretically possible to use the source-coupled FET circuit in a Van der Pol oscillator. It has been shown experimentally (see below) that the circuit has an almost cubic input characteristic.

At low frequencies, the reactance of the feed-back capacitor must be accounted for, and we must replace R_o in equation (8) by $R_o + 1/j\omega C$. It may then be shown that the circuit appears as a negative resistance in series with a negative capacitance. At high frequencies many capacitance effects must be considered and it may be shown that the circuit appears as a negative resistance in series with a capacitive reactance. This has been demonstrated[4] for the analogous twin-triode cathode-coupled circuit.

3. Input Characteristic

The input characteristic obtained experimentally is shown in Fig. 4 for the prototype circuit in Fig. 5. A standard computer programme was used to obtain a polynomial fit given by :

$$i = 0.244\times10^{-11} - 0.463\,v - 0.522\times10^{-12}\,v^2 + 0.0361\,v^3 \text{ mA}$$

Evidently the first and third terms are negligible giving the cubic :

$$i = O.463\,v + 0.0361\,v^3 \text{ mA.} \quad \ldots(11)$$

This approximation fitted the observed characteristic to within 3% error in the negative resistance region, and is shown by the broken lines in Fig. 4. The negative resistance in the vicinity of v = O is 1/.463 i.e. 2.16 kΩ and is very close to the value 2.2 kΩ predicted by equation (10). Fig. 6 shows the variation of input negative resistance with source resistance R_s and the agreement between observed and predicted values is excellent over a wide range of R_s.

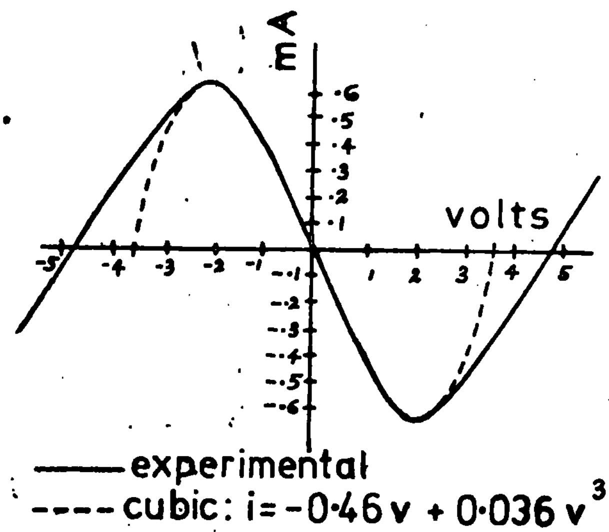


Fig.4

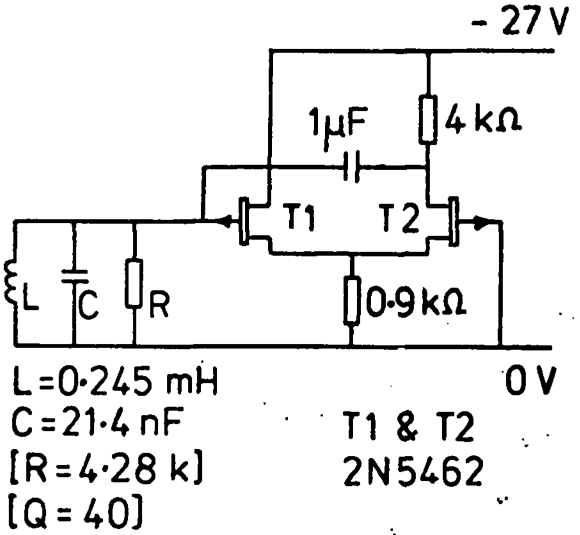


Fig.5

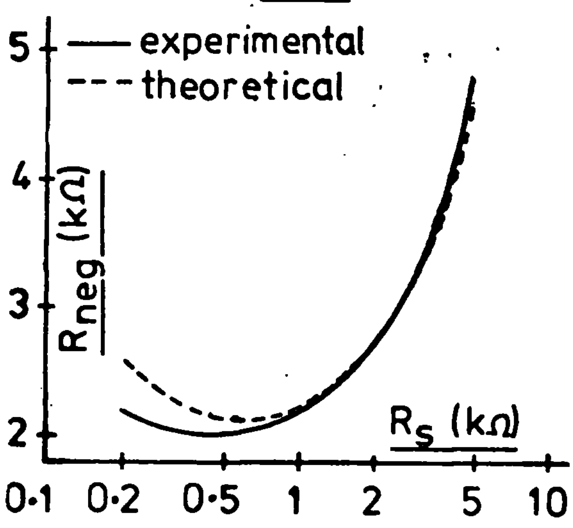


Fig.6

The negative resistance was measured by displaying the characteristic on a cathode ray oscilloscope and decreasing a positive resistance connected in parallel with the non-linear element, until the negative resistance portion of the characteristic became horizontal. In order to estimate g_{m1} and g_{m2} the source currents of both FET's were monitored for each setting of R_s and a separate experiment was performed (vide Appendix) to measure g_m as a function of source current (dependence on drain voltage was negligible within the working range) for each FET.

4. Free Running Oscillator

The constants of the tank circuit used are given in Fig. 5 for which $f_o = \frac{1}{2\pi\sqrt{(LC)}} = 69.95$ kHz. The Q-factor was measured to be 40 corresponding to a value of R equal to 4.28 kΩ. These values satisfy the inequality (5) according to $107 < 2160 < 4281$. The constant α is calculated to be 0.0245 which is much less than unity as required.

The free running amplitude E was measured to be 2.58 V, the calculated value being $E = \frac{2}{\sqrt{\Delta}} = 2.9$ V showing a discrepancy of 11%. The free-running frequency varied from 72.971 kHz to 69.738 kHz as R_L was taken from zero upto 9 kΩ as shown in Fig. 7. (A 2.5 mH inductor was inserted in the drain line of T2 for this measurement). Change in Rs and d.c. supply voltage had little effect on frequency. The output voltage variation with change in d.c. supply is shown in Fig. 8. The output is practically constant at 1.83 V (r.m.s.). However, the output varied considerably with R_L and R_S as shown in Fig. 9, (again with a 2.5 mH inductor in the drain line of T2).

With a coil of less than 10 μH resonant with its self-capacitance, good oscillations were achieved upto 8.6 MHz, with a sinusoidal output of over 2.5 V r.m.s. The drain load consisted of only the 2.5 mH inductor.

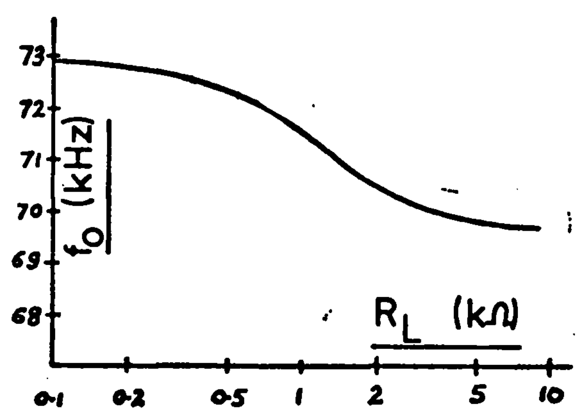


Fig.7

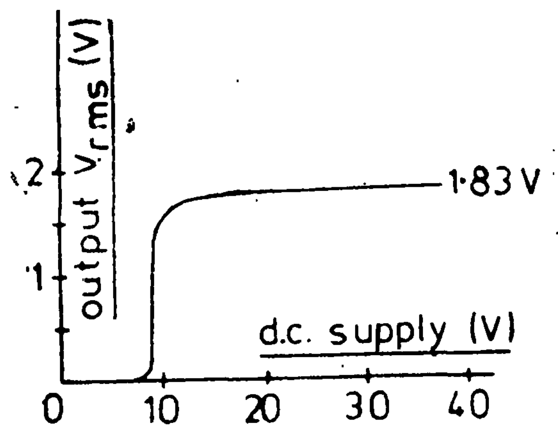


Fig.8

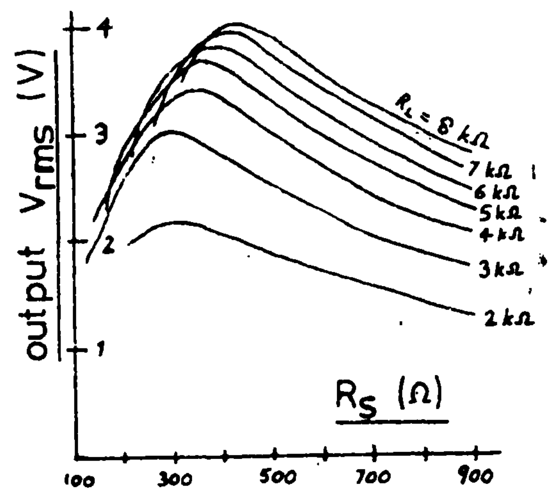


Fig.9

With a coil of less than 10–H resonant with its self-capacitance, good oscillations were achieved upto 8.6 MHz, with a sinusoidal output of over 2.5 V r.m.s. The drain load consisted of only the 2.5 mH inductor.

5. Conclusion

The oscillator circuit described should be noted for its output voltage and frequency stability with supply voltage variation. The high frequency limit is set by the FET performance and provided suitable FET's and tank circuits are used, operation could be extended to the VHF region and above. Various quantities of interest such as the amplitude and frequency of oscillations and the percentage of harmonics in the output can be accurately calculated at the design stage.

References

1. B. Van der Pol, "*Forced Oscillations in a circuit with Non-linear resistance*", Phil. Mag., Vol. 3, 1927.

2. T. L. Hooper, "*A Calculable Locked Oscillator*", Inst J. Elec. Eng. Educ., Vol. 1, 1963, p. 133.

3. W. A. Gambling, et al, Int. J. Elec. Eng. Educ., Vol. 2, 1965, p. 573.

4. P. G. Sulzer, "*Cathode-coupled Negative-resistance Circuit*", Proc. I.R.E., Vol. 36, Aug. 1948, pp. 1034–1039.

Bibliography

1. D. F. Lawdon, Mathematics of Engineering Systems, Methuen, London, 1954, p. 315.

2. W. J. Cunningham, Introduction to Non-linear Analysis, McGraw-Hill, New York, 1958, p. 140, 213.

3. J. D. Parsons and F. V. C. Mendis, "*Negative-admittance Circuit with an approximately Cubic characteristic*", I.E.E.E. Trans. circuit theory, vol. CT-19, No. 1, Jan. 1972, p. 105.

Appendix

The measurement arrangement is shown in Fig. 10, with an oscilloscope being used as the detector D. The current is varied by changing the bias, e_g, and at each setting, R is adjusted for zero indication in D, when it can be shown that gm = 1/R.

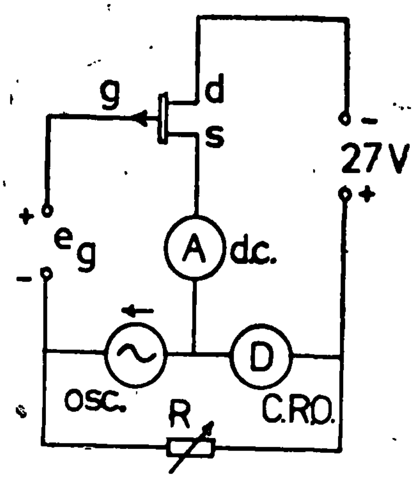


Fig.10