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Pipe Diameter Optimization using Genetic Algorithm-A Case Study

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Abstract: A systematic study on the application of genetic algorithm (GA) in pipe diameter optimization was carried out. The aim of the study was to develop a genetic based programme and apply it to a case study, distribution system of Weligama water supply scheme. Then the performance of Genetic algorithm was compared with non linear programming (NLP).

It was found that the genetic algorithm method using the standard three operators is particularly effective in finding global optimal solution for the case study network. It should be noted that the genetic algorithm technique is ideally suited to discrete problems such as selection of commercially availably pipe sizes.

Keywords: Genetic Algorithm, Global Optimum, Pipe Network

1. Introduction

Optimization concept is intrinsically tied to human intuition. A desire for perfection or a search for the best is naturally demanded. As a vital part of the water supply systems, water distribution networks represent one of the largest infrastructure assets of water industrial society. In water supply design point of view, whether the ultimate goal is to assure for more reliable supply of water with respect to space and time, the achievements of economic advantage are more meaningful, because funds available for the development of new infrastructure become increasingly scarce. As a result, pipe network optimization becomes a field of study in its own right.

Genetic algorithm is proposed as one possibility of optimum search algorithms. Since the early development by HOLLAND in 1975[4], the algorithms have been theoretically and empirically proved to be a very effective and robust search method. Advantages of genetic algorithm are due to its simplicity, straightforwardness and commitment for better and better solutions. Genetic algorithms were employed in a wide-ranging of pipe network optimization problems and the capability of genetic algorithms were evident in various research studies [7], [8], [12].

2. Literature Review

Golberg [2] examined the mechanics, power and application of a genetic algorithm to a steady state optimization of a serial liquid pipeline. The objective was to minimize power, while supplying a specified flow and maintaining allowable pressures. Simple three operator genetic algorithm was applied to the on-off control of 40 pumps in 10 pump stations along a serial pipeline. The pipeline model has been programmed in Pascal. Constraints have been adjoined to the problem with an exterior penalty function. The genetic algorithm was run with 100 population size. The probability of crossover and probability of mutation were taken as 0.7 and 0.01. It was found that GA gives near optimal operation pump schedules quickly.

Savic and Walters [7] described the development of the computer model GANET for the least cost design of water distribution networks and further more demonstrated the superior capability of GA against other optimization techniques. Least cost pipe network design was applied to a sample network with a single source at a 210m fixed head and eight pipes arranged in two loops. A binary string made of eight sub strings was used for coding the problem.

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The evaluation function determined the cost of a solution. Rank selection was used for reproduction. Ultimate results showed that the GA produces good design.

Meier [5] described the use of a GA to optimize the sampling design. GA was applied to a network model for a small town in Ohio. Probability of crossover and mutation was taken as 0.7 and 0.02 respectively. GA was shown to perform extremely well, matching the optimal solution produced by complete enumeration in a series of validation tests.

3. Study Area

GA is applied to a water supply pipe network in Weligama area in Matara district which has been constructed in 1991. This water supply scheme covers about 13km² area of Matara District in Southern province. Designed demand is 10,950 m³/day by 2011. In this study, only the distribution system is analysed, where the water flows under gravity from a reservoir of 910 m³ capacity at Kottegoda. It consists of pipe lines of PVC Type 600 of several diameters ranging from 50 -250mm. Information on Elevations, pipe lengths, and demand were collected.

4. Methodology

4.1 Theoretical background-Genetic Algorithms

Genetic algorithms are search algorithms based on Darwin's theory of survival of the fittest [2]. Through the three basic operations; reproduction, crossover and mutation, genetic algorithms search for optimum solution. Decision variables should be represented in a appropriate coding scheme. The search begins with this group of strings. Every string in a population is processed concurrently generation-by-generation under controlling of genetic operations. Reproduction operator selects the effective strings from population relying on their objective functions or fitness values. Effective strings are then controlled to breed for new generation by crossover. Hence, new strings are introduced by gathering information from fittest pairs of string or chosen mate. Mutation operator simulates a natural adaptation by transmuting some bits in string to prevent irrecoverable losses of data. Penalty function is introduced to prevent searching in constrained violating domain. Termination criteria are predefined to stop the search algorithms.

4.2 Pipe network Optimization

Water distribution network optimization is a procedure to seek for design variables of a network to its goal of optimality, within the constraints. Objective function or the cost function should be formulated whose least values is searched in an optimization procedure.

4.2.1 General pipe network Optimization Problem

Objective function of the general pipe network optimization problem can be shown as;

$$\text{Minimize } F(D) = \sum_{i=1}^{N} c(D_i, L_i) \qquad (1)$$

Where $c(D_i,L_i)$ is the cost of the pipe with the diameter D_i and the length L_i, and N is the total number of pipes in the system.

This objective function is minimized under the flowing constraints:

For each junction,

$$\sum Q_{in} - \sum Q_{out} - \sum Q_e = 0 \qquad (2)$$

Where

Q_{in}, Q_{out} and Q_e are flow into the junction , out of the junction and external inflow or demand at the junction node respectively.

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For each loop,

$$\sum h_f - \sum E_p = 0 \qquad (3)$$

Where h_f and E_p are head loss and energy put into the liquid by a pump.

For each node,

$$H^{\max} \ge H_j \ge H^{\min} \qquad ; j = 1, \ldots n \qquad (4)$$

Where H_j, $H^{\max}$ and $H^{\min}$ are head at node j, maximum and minimum allowable head respectively.

For each pipe,

$$V^{\max} \geq V_j \geq V^{\min} \quad ; j = 1,....n \qquad (5)$$

Where V_j, $V^{\max}$ and $V^{\min}$ are velocity at pipe j, maximum and minimum allowable flow velocity respectively.

5.2 Computer Program of Three Operator Genetic algorithm

Microsoft visual C++ version 6.0 was used and the programme was written in C language. Elevation, nodal demand, available pipe diameter sizes and their unit prices were the input data files.

Binary coding is used to code the decision variable. GA generates the initial population of solutions (of say, size N=100) using a random number generator. Number of population

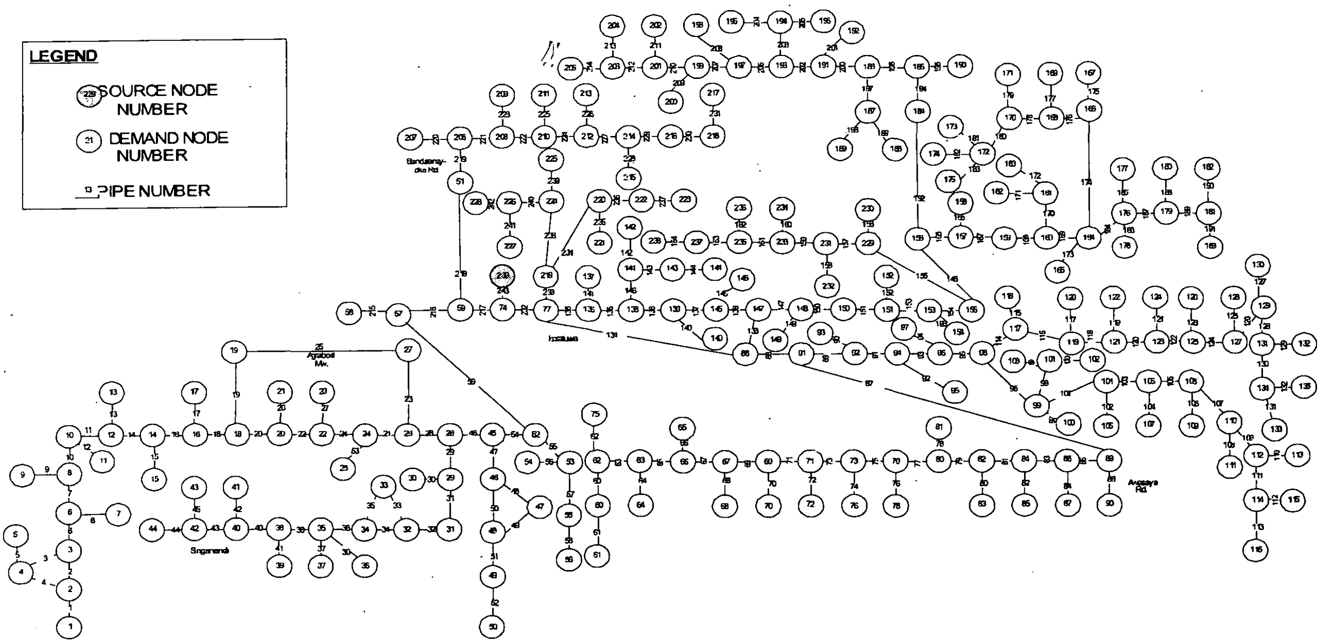

Figure 1- Layout of the pipe network

5. Analysis

5.1 Assumptions and design criteria

The following assumptions were made in hydraulic design.

- The minimum service pressure head in the distribution system should not be less than 10m and the maximum service pressure head should not higher than 50m.
- The Hazen Williams coefficient C is assumed to be 130.
- Minimum and maximum flow velocities are taken as 0.6m/s and 1.6m/s respectively.
- The flow state is assumed to be steady for the whole network.
- The maximum head loss allowance should not exceed 10 meters per kilometre length of pipe line.

should be in the range of 30-200[3]. Options available for existing pipes are arranged in terms of increasing cost and the binary string representation of available pipe sizes is given in the Table 1.

To evaluate the fitness of a string , the string must be converted into a design solution by mapping a string value onto a variable value. Assume one bit of one substring for one variable is represented as a_1, a_2...... a_n, the value of the substring (corresponding diameter index) can be calculated as

$$S_i = \sum_{n=1}^{Nbit} a_n b^{(Nbit-n)} \qquad (6)$$

Where $Nbit$ is the number of bits in the substring and b is equal to 2.

Table 1- Binary string representation, corresponding diameter index of available pipe sizes and unit prices

Binary substring coding pipe size	Dia. index	Available pipe diameters (mm)	Unit price ($/m)
000	0	47	0.45
001	1	58	0.70
010	2	69	0.92
011	3	83	1.30
100	4	102	1.95
101	5	148	4.10
110	6	208	9.00
111	7	280	13.00

Network cost is calculated for each population by taking the summation of product of pipe length and unit cost of pipe.

Two governing equations used are continuity equation and Hazen William's head loss equation [9].

$$h_f = \frac{10.675 L_i * Q_i * | Q_i |^{0.852}}{C_i^{1.852} * D_i^{4.8704}} \tag{7}$$

Where

L_i = Length of the pipe

D_i = Diameter of the pipe

Q_i = Flow through the pipe

C_i = Hazen William coefficient

The head loss equation is expressed for each pipe and the continuity equation is applied at each node throughout the water supply distribution system. The flow and head loss conditions are determined for each pipe in the loops by solving the two equations iteratively.

Knowing the flow rate through each pipe, velocities can be calculated.

$$V_i = \frac{Q_i}{\pi\left(D_i^2/4\right)} \tag{8}$$

Knowing the head loss and elevation at each node, pressure head can be calculated.

The total penalty cost is taken as the summation of the product of the pressure head constraint violations (P_1) times a specified penalty multiplier (R_1) and the product of the velocity constraint violations (P_2) times a specified penalty multiplier (R_2). R_1 is a measure of the worth per metre attributed to pressure heads below and above the allowable minimum and maximum pressure heads. R_2 is a measure of the worth per metre attributed to velocities below and above the allowable minimum and maximum velocities.

Fitness of each population is calculated as the summation of network cost and the total penalty cost [8].

$$f_i = F(D) + R_1 P_1 + R_2 P_2 \tag{9}$$

Where F(D) = $\sum_{i=1}^{N} c(D_i, L_i)$, $c(D_i, L_i)$ is the cost of the pipe with the diameter D_i and the length L_i, and N is the total number of pipes in the system.

Then the GA generated new members of the new generation by tournament selection method, where two or more chromosomes are randomly picked from the population, and the best one selected for further genetic processing.

The uniform crossover operator is practiced next. It is the partial exchange of bits between two parent strings to form two offspring strings. A crossover probability (p_c) is assumed between (0.5-1.0). The GA randomly picks two strings from the new population. A uniformly distributed random number is then generated in the range 0.0 and 1.0. The GA applies the crossover operator if the random number is less than the probability of crossover.

To ensure no important genetic material is lost, Mutation operator is applied. A probability of mutation (p_m) is assumed between 0.01-0.1. The GA considers each string bit by bit in the new generation formed as a result of reproduction and crossover. A uniformly distributed random number in the range of 0.01 to 0.1 is generated. GA applies the mutation operator if the random number is less than the probability of mutation. Otherwise the GA does not apply mutation to that particular bit. The mutation operator changes the value of the bit to the opposite value. (i.e. a 0 to 1 or a 1 to 0).This procedure repeats until the optimal value is reached.This solution is tested with the Loop hydraulic simulation model.

Random number seed	Least cost (M $)
0	0.123
100	0123
500	0.123
1000	0.144
5000	0.123

5.4 Non-linear programming.

The case study problem is solved using the nonlinear optimization package, "GAMS". The General Algebraic Modelling System (GAMS) language is formally similar to commonly used programming languages. GAMS analysis is carried out independently of genetic algorithm analysis.

A cost function for pipes is fitted (Figure 2) and it is given below (Equation 10).

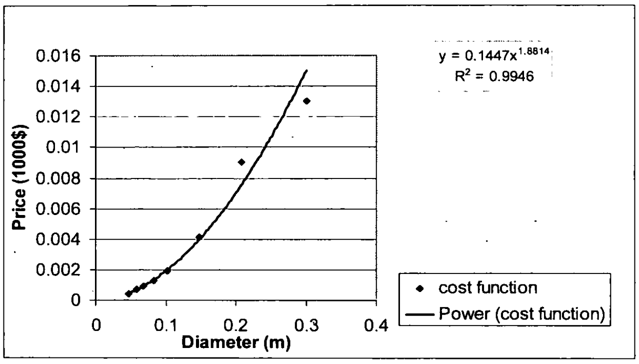

Figure 2 - cost function

$$= 0.144D^{1.8814} \qquad C \qquad (10)$$

Where C= cost of pipes
D= diameter of pipe in m

Constraints, same as (2)-(5) were included in the model.

6. Results and Discussion

6.1 Searching of global optimality

Five runs for the case study network problem using different random number seeds were carried out to check whether the global optimality is achieved. A maximum of 50,000 functional evaluations were permitted for each run. The lowest cost achieved in each run is presented in table 2. The least cost was found at the first of 300 generation.

Table 2- Results of genetic algorithm runs for 10,000 evaluations

6.2 Parameter selection

Twenty additional runs were performed to assess the sensitivity of the technique to parameter settings. The quality of solution obtained from GA depends upon several important parameters, such as, population size, probability of crossover (p_c) and probability of mutation (p_m). Each parameter was examined during testing process, in the way that only the testing parameter was changed while the others were kept constant. Table 3 shows the GA parameters and lowest cost results.

Table 3-GA Parameters and Cost Results

Run no.	Pop. size (n)	Prob. of crossover (p_c)	Prob. of mutation (p_m)	Lowest cost (M $)
1	30	0.7	0.04	0.168
2	50	0.7	0.04	0.168
3	100	0.7	0.04	0.144
4	200	0.7	0.04	0.123
5	250	0.7	0.04	0.131
6	300	0.7	0.04	0.168
7	200	0.7	0.01	0.152
8	200	0.7	0.02	0.152
9	200	0.7	0.03	0.152
10	200	0.7	0.04	0.123
11	200	0.7	0.05	0.152
12	200	0.5	0.04	0.123
13	200	0.7	0.04	0.123
14	200	1.0	0.04	0.123

6.2.1 Effect of Population Size

Several population sizes were examined. The best number for the population size is taken as 200, since it gives the lowest cost.

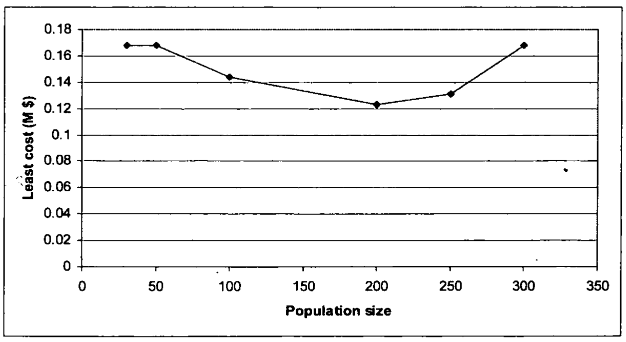

Figure 3- The effect of population size

6.2.2 Effect of Probability of Crossover

The probabilities of crossover of 0.5, 0.7, and 1.0 were examined and the effect is shown in the Figure 4. It shows no effect of probability of cross over.

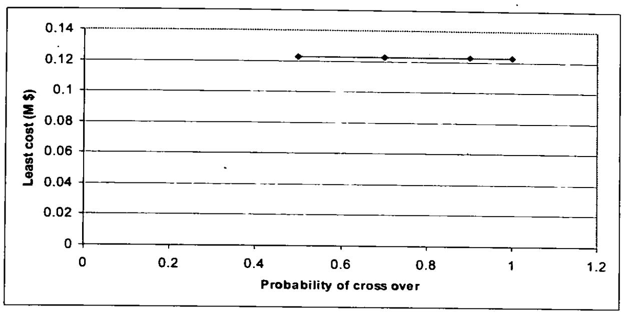

Figure 4-The Effect of Probability of Crossover

6.2.3 Effect of Probability of Mutation

Several probabilities of mutation were examined and the effect is shown in the Figure 5. The best value for the probability of mutation was taken as 0.04 ,since it gives the minimum cost.

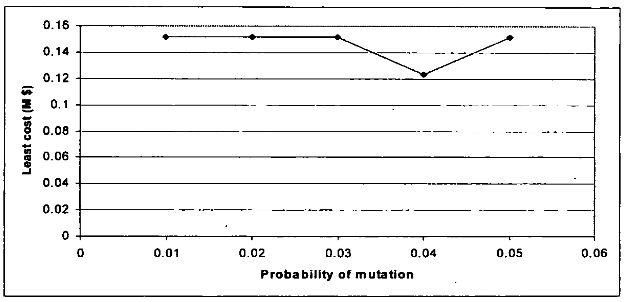

Figure 5-The Effect of Probability of Mutation

6.2.4 Effect of Penalty Multipliers

The effect of Penalty multipliers, R_1 and R_2 were tested and their effect on the fitness is shown in figure 6 and 7.

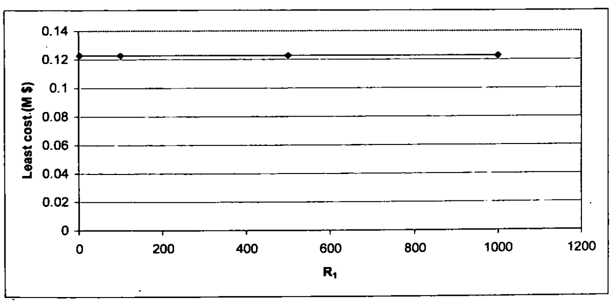

Figure 6 - The effect of R_1

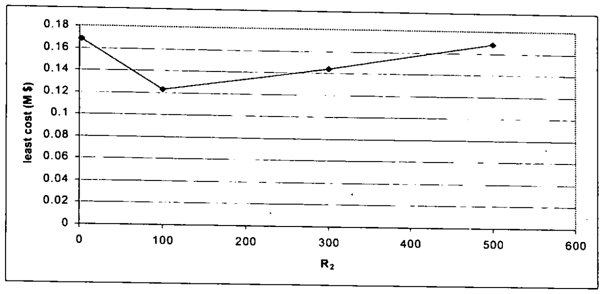

Figure 7 - The effect of R_2

6.3 Final Result of GA

The run characteristics, criteria formulation for GA and the final result of GA are given in Table 4. GA run took 270 CPU min on a Pentium 4, CPU 1600 MHz, RAM 130,544 KB computer to search the least cost of the pipe network as Million US $ 0.123. Optimum diameter given by the GA are shown in Table 5.

The solution from the GA was simulated with the help of LOOP hydraulic simulation model.

Table 4- Results of GA

Item	Results
Population size	200
Execution time in minuets with Pentium IV 1600Mhz,Ram 130544kb	270
Number of alleles per gene	3
Length of chromosome	729 (=3*243)
Probability of crossover	0.7
Probability of mutation	0.04
Type of selection	Tournament selection
Type cross of over	Uniform crossover
Type of mutation	Uniform mutation
Best fitness (M $)	0.123

6.4.3. Result of Non Linear Program

It gave the least cost as M $ 0.11. The optimization run took 3.5CPU min on a Pentium 4, CPU 1600 MHz, RAM 130,544 KB computer. The solutions given by NLP are shown in Table 5. Many of the values given as pipe diameters could not found in the market. Therefore it is needed to round up the pipe sizes to the nearest available diameter manually and run Loop software to ensure the rounded solution satisfies the constraints. Then the result came similar to that of genetic algorithm.

Table 5- Solutions given by GA and NLP

Pipe	Diameter (m)		Pipe	Diameter (m)		Pipe	Diameter (m)		Pipe	Diameter (m)		Pipe	Diameter (m)	
No.	GA	NLP	No.	GA	NLP	No.	GA	NLP	No.	GA	NLP	No.	GA	NLP
1	47	47	51	69	60	101	102	99	151	208	208	201	47	47
2	47	47	52	47	50	102	47	47	152	47	47	202	148	148
3	83	74	53	47	47	103	102	102	153	208	208	203	69	67
4	47	47	54	208	202	104	47	47	154	208	47	204	47	47
5	47	47	55	58	55	105	102	100	155	102	199	205	47	47
6	102	91	56	47	47	106	47	47	156	58	102	206	148	146
7	102	100	57	58	56	107	83	78	157	83	83	207	102	102
8	47	47	58	47	47	108	47	47	158	47	47	208	69	69
9	47	47	59	208	205	109	69	66	159	69	67	209	47	47
10	102	102	60	47	47	110	47	47	160	47	47	210	102	101
11	148	120	61	47	47	111	69	62	161	58	56	211	47	47
12	47	47	62	47	47	112	47	47	162	47	47	212	83	82
13	47	47	63	58	55	113	47	47	163	47	47	213	47	47
14	148	125	64	47	47	114	83	79	164	47	47	214	69	65
15	47	47	65	69	62	115	47	47	165	148	133	215	47	47
16	148	130	66	47	47	116	83	83	166	47	47	216	208	179
17	47	47	67	83	78	117	47	47	167	148	126	217	208	208
18	148	142	68	47	47	118	83	74	168	148	146	218	83	78
19	47	47	69	102	102	119	47	47	169	148	145	219	83	79
20	148	143	70	47	47	120	83	83	170	47	47	220	47	47
21	148	148	71	102	99	121	47	47	171	47	47	221	83	83
22	148	142	72	47	47	122	83	83	172	47	47	222	69	69
23	47	47	73	102	100	123	47	47	173	47	47	223	47	47
24	148	148	74	47	47	124	69	66	174	102	100	224	69	64
25	47	47	75	148	144	125	47	47	175	47	47	225	47	47
26	47	47	76	47	47	126	69	69	176	83	81	226	47	47
27	47	47	77	148	148	127	47	47	177	47	47	227	58	51
28	148	136	78	47	47	128	58	55	178	69	65	228	47	47
29	102	102	79	148	144	129	47	47	179	47	47	229	58	52
30	47	47	80	47	47	130	58	55	180	47	47	230	47	47
31	102	102	81	148	148	131	47	47	181	47	47	231	47	47
32	102	102	82	47	47	132	47	47	182	47	47	232	280	262
33	47	47	83	148	136	133	102	100	183	47	47	233	102	102
34	102	100	84	47	47	134	208	200	184	83	81	234	69	67
35	47	47	85	148	126	135	208	203	185	47	47	235	47	47
36	102	101	86	47	47	136	208	208	186	47	47	236	47	47
37	47	47	87	148	133	137	208	202	187	47	47	237	47	47
38	83	82	88	208	202	138	208	208	188	47	47	238	83	77
39	47	47	89	208	208	139	208	178	189	47	47	239	47	47
40	69	65	90	47	47	140	47	47	190	47	47	240	69	66
41	47	47	91	208	208	141	47	47	191	47	47	241	47	47
42	47	47	92	47	47	142	47	47	192	148	140	242	47	47
43	47	47	93	208	200	143	47	47	193	47	47	243	280	280
44	47	47	94	47	47	144	47	47	194	148	140			
45	47	47	95	148	145	145	47	47	195	47	47			
46	208	200	96	148	148	146	58	58	196	148	136			
47	102	102	97	47	47	147	208	199	197	69	66			
48	83	80	98	102	98	148	208	185	198	58	58			
49	58	58	99	47	47	149	47	47	199	47	47			
50	58	57	100	58	58	150	208	190	200	148	136			

7. Conclusions

Non linear optimization is the fastest of the two optimization techniques applied to the case study network. But one needs to fit a cost function to the discrete pipe cost data and this may leads to inaccuracies. On the other hand it is needed to round the continuous solution found either up or down to the nearest discrete pipe sizes manually. Additional computer runs are needed in Loop software to ensure that the rounded solution actually satisfies pressure and velocity constraints. Therefore, although non linear optimization is quick, reaching to a required discrete pipe solution takes days.

The computer time required in genetic algorithm is comparatively long. It is found that the genetic algorithm method using the standard three operators is effective in finding the optimal solution, especially discrete problem like finding the optimum pipe diameters, where commercially available pipe sizes are discrete.

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