# NETWORK SYSTEM MODELING AND THE OPTIMUM ROUTING PROCEDURE

by

**Dr. Shahane de Costa and Dr. Y. Tsunematsu**

## Abstract

For the purpose of facilitating direct simulation of the channel network system on computers, one of the requirements, which is the optimum routing procedure of channel network systems has been discussed and a simple four step optimum routing procedure is proposed. The channel network system is then simulated using a Graph-Theoretic frame work and a set of algorithms are developed for direct channel network system modelling and not piece wise channel modeling. This is then applied to a model network.

## 1.0 Introduction

Channel network systems generally take complex structures specially in the uplands where there are rapid topographic changes. Nevertheless these network systems have to be simulated on computers to facilitate channel routing. However one of the problems associated with this is the optimum routing procedure. Another problem is the direct modeling of the whole network system compared to the piece by piece modeling of each reach.

Considering these problems, here in this paper a new, simple 4 step optimum routing procedure for channel network systems is proposed. A set of algorithms are also developed through an analytical method for the purpose of direct modeling of the network system.

## 2.0 Optimum Routing Procedure

In a complex network system as it could tend to be difficult to locate the relative upstream point, a necessity exists for a standard method where a channel network system could be numbered in order from up stream to down stream. In this respect at present we have research done by Takasao and Shiba [1]. There is also a method proposed by Minijiao and Koike [2].

Analyzing Takasao and Shiba's research it could be seen that there is no necessity for two independent persons to route the same network system in the same manner which means that this method lacks uniformity. It is also well known that Minijiao and Koikes routing procedure is highly complex and long, and there by requires a separate study in itself. Therefore, here a new simplified four step procedure is given to find the optimum routing procedure for channel network systems. The features of this in comparison to the existing methods are its simplicity and its uniformity.

The optimum routing procedure is as follows.

1. First, all the channels are ordered using the strahler method [3]. Here, for all channels that do not have an upstream confluence point the order number Or=1 is given. Now going down the ordered channels if two or more channels confluence then the order of the new channel is increased by one (ie. Or=Or+1). If not the same order continues. In mathematical form it would be as follows.

$$\overset{d}{Or} = \max(\overset{u}{Or_1},\ \overset{u}{Or_2} + 1 \,.........\, \overset{u}{Or_{mc}} + 1)$$

where, $(Or_1 > Or_2 \,.........., Or_{mc})$

d : Downstream

u : Upstream

mc : Maximum number of ordered channels

**Dr. Shahane de Costa,** *graduated in Civil Engineering from the University of Moratuwa in 1987. Then he obtained a M.Sc. in Environmental Engineering in 1991, and the degree of Doctor of Engineering in 1994 from the University of Hiroshima. At present he is working at Penta Ocean Construction Co. Ltd. which is engaged in the Colombo Port Expansion Project.*

**Assoc. Prof. Y. Tsunematsu,** *obtained the B.Sc. in Civil Engineering from the University of Hiroshima and the degree of M.Sc. & Doctor of Engineering from the University of Kyoto in 1972 and 1975 respectively. At present he is lecturing at the University of Hiroshima & is attached to the Hydraulics & River Engineering Laboratory.*

2. (a) Next the node/edge at the down stream most point which also has the maximum order number is selected as the pivotel node/edge and then going upstream from there giving preference to the left hand most node/edge the node/edge which has the next highest order number if not the same is selected.

   (b) Now taking the newly selected node/edge as the pivotel node/edge step 2(a) is repeated.

3. Step 2 (a) and 2 (b) are repeated until the source node/edge is reached.

4. (a) Once the source node/edge is reached, return back to the last pivotel node and steps 2 and 3 are repeated deleting the already reached source node/edge.

   (b) Steps 2, 3 and 4(a) are repeated until all the nodes/edges are deleted.

   (c) The order in which the nodes/edges are deleted is selected as the optimum routing procedure. For example the first node deleted is given the number one and its taken up first in order for routing. The edges are given the same number as its upstream node.

Therefore following the above steps the optimum routing procedure for channel network systems could be obtained. An example is shown in Section 5.

## 3.0 System Modeling [4], [5]

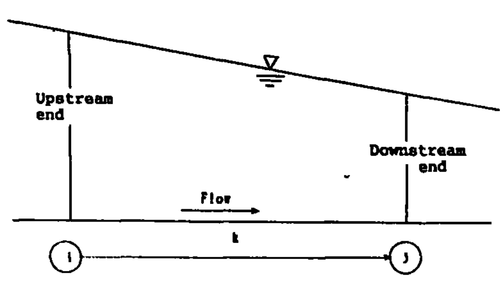


**Figure 1 - Sketch of directed graph for a single reach**

The channel network system is assumed to be composed of channels, confluence points etc. A discrete graph of the system is defined for organising basic equations in a graph theoretical frame work to facilitate direct implementation on computers [6]. Each channel or reach is called an "edge" with a direction arbitrarily defined in the direct graph. The two ends of the edge are called "nodes" where a confluence point may exist. Fig. 1 represents a sketch of a directed graph for a single reach. The number of edges and nodes are Ne and Nn respectively. The optimum numbering procedure of edges and nodes are described in section 2.

The incidence matrix or the connection matrix is denoted by D.

$$\text{where, } D[D_{ik}]: \left\{ \begin{array}{l} D_{ik} = 1 \text{ ; } k^{th} \text{ edge is oriented away from } i^{th} \text{ node} \\ D_{ik} = -1 \text{ ; } k^{th} \text{ edge is oriented towards } i^{th} \text{ node} \\ D_{ik} = 0 \text{ ; } k^{th} \text{ edge is not incident at } i^{th} \text{ node} \end{array} \right\} \quad (1)$$

in which $D_{ik}$ =(i, k) element of matrix D (i=1, 2,.... Nn:K=1,2....Ne) (Nn: number of nodes; Ne: number of edges).

$D_{ik}$ is set to unity if the $K^{th}$ edge is incident in the $i^{th}$ node and if not $D_{ik}$ is set to zero. The sign plus or minus is decided upon the orientation of the $K^{th}$ edge towards the ith node.

If the incidence matrix of the $i^{th}$ node is separated to $D_i^-$ and $D_i^+$ where, the edges oriented away is denoted by $D_i^+$ and the edges oriented towards is denoted by $D_i^-$, then the equation of the incidence matrix at that node would be as in eq. (2).

$$D_i = D_i^+ + D_i^- \qquad (2)$$

At the source which are the upper most nodes $D_i^-$ will be zero as there are no edges oriented towards it while at the sink which is the down most node $D_i^+$ will be zero as there are no edges oriented away from it.

Using the theory of continuity of discharge at a given node, will result in eq. (3). Here $q_i$ is the boundary condition.

$$D_i^+ Q_i^+ + D_i^- Q_i^- = q_i^+ \qquad (3)$$

Where $Q_i^+$ is the discharge matrix leaving the $i^{th}$ node and $Q_i^-$ is the discharge matrix entering the $i^{th}$ node.

The incorporation of this theory to the general discharge equation or the equation developed in previous research [7], could be done as follows.

First the discharge matrix entering and leaving a node should be obtained.

The general discharge equation which is,

$$A_{J-1}\,Q_{J-1}^{n+1} + B_J\,Q_J^{n+1} = C_{J-1} \qquad (J=2, \ldots\ldots m)$$

$A_{J-1}$, $B_J$ : Coefficients which incorporate discharge values at the time step n.

m : Number of cross sections.

could be rewritten as in the submatrix form of eq. (5) such that $a_1$ ........, $a_9$ $z_1$, $z$, , $z_m$, $b_1$, $b$, $b_{m-1}$, and so on represent that portion demarcated by the bold line of eq. (4).

$$\left[\begin{array}{c|ccc|c} A_1 & B_1 & & & 0 \\ \hline \vdots & \ddots & \ddots & & \vdots \\ 0 & & A_{J-1} & B_J & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ \hline 0 & & & A_{m-1} & B_m \end{array}\right] \left[\begin{array}{c} Q_1^{n+1} \\ \hline Q_2^{n+1} \\ \vdots \\ Q_{m-1}^{n+1} \\ \hline Q_m^{n+1} \end{array}\right] = \left[\begin{array}{c} C_1 \\ \hline C_2 \\ \vdots \\ C_{m-2} \\ \hline C_{m-1} \end{array}\right] \qquad (4)$$

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} \begin{pmatrix} z_1 \\ z \\ z_m \end{pmatrix} = \begin{pmatrix} b_1 \\ b \\ b_{m-1} \end{pmatrix} \qquad (5)$$

Expressing eq. (5) as eqs. (6), (7) and (8)

$$a_1 z_1 + a_2 z = b_1 \qquad (6)$$

$$a_5\, z = b \qquad (7)$$

$$a_8 z + a_9 z_m = b_{m-1} \qquad (8)$$

and as $Z = a_5^{-1} b$, eq. (5) could be simplified as follows

$$\begin{pmatrix} a_1 & a_2 a_5^{-1} b \\ a_8 a_5^{-1} b & a_9 \end{pmatrix} \begin{pmatrix} z_1 \\ z_m \end{pmatrix} = \begin{pmatrix} b_1 \\ b_{m-1} \end{pmatrix} \qquad (9)$$

This equation could be expressed in a general form as,

$$[\,C^+ \; C^-\,] \begin{bmatrix} Q^+ \\ Q^- \end{bmatrix} = [\,G\,] \qquad (10)$$

where,

$$[C^+] = \begin{bmatrix} a_1 \\ a_8 a_5^{-1} b \end{bmatrix}, \quad [C^-] = \begin{bmatrix} a_2 a_5^{-1} b \\ a_9 \end{bmatrix} \quad [G\,] = \begin{bmatrix} b_1 \\ b_{m-1} \end{bmatrix}$$

Here, $Q^+ = Q_1^{n+1}$

$Q^- = Q_m^{n+1}$

for the edge K where, K=1,2......Ne

This is then expanded for the whole network system as explained below to facilitate direct channel network system modeling and not piecewise channel modeling.

$$\begin{bmatrix} C_1^+ & C_1^- \\ \vdots & \vdots \\ C_{Ne}^+ & C_{Ne}^- \end{bmatrix} \begin{bmatrix} Q_1^+ & \cdots & Q_{Ne}^+ \\ Q_1^- & \cdots & Q_{Ne}^- \end{bmatrix} = \begin{bmatrix} G_1 & & 0 \\ & \ddots & \\ 0 & & G_{Ne} \end{bmatrix} \qquad (11)$$

Where the suffix 1..... Ne refers to the edge number.

This could be rewritten in the vector form as Eq. (12).

$$[\,\bar{C}^+ \; \bar{C}^-\,] \cdot \begin{bmatrix} \bar{Q}^+ \\ \bar{Q}^- \end{bmatrix} = [\bar{G}] \qquad (12)$$

Where,

$$[\bar{C}^+] = \begin{bmatrix} C_1^+ \\ \vdots \\ C_{Ne}^+ \end{bmatrix} \quad [\bar{C}^-] = \begin{bmatrix} C_1^- \\ \vdots \\ C_{Ne}^- \end{bmatrix} \quad [\bar{G}] = \begin{bmatrix} G_1 & & 0 \\ & \ddots & \\ 0 & & G_{Ne} \end{bmatrix}$$

$$[\bar{Q}^+] = [Q_1^+ \ldots\ldots Q_{Ne}^+]$$

$$[\bar{Q}^-] = [Q_1^- \ldots\ldots Q_{Ne}^-]$$

Therefore since the discharge matrix leaving and entering the node has been obtained the general equation for the network system could be expressed using eqs. (3) and (12) as follows.

$$\begin{bmatrix} \bar{C}^+ & \bar{C}^- \\ & \\ \bar{D}^+ & \bar{D}^- \end{bmatrix} \begin{bmatrix} \bar{Q}^+ \\ \\ \bar{Q}^- \end{bmatrix} = \begin{bmatrix} \bar{G} \\ \\ \bar{q} \end{bmatrix} \qquad (13)$$

Where

$$[\mathbf{q}] = \begin{bmatrix} q_1 \\ \vdots \\ q_{Nn} \end{bmatrix}$$

and $\bar{D}^-$ and $\bar{D}^+$ indicates (Nn x Ne) in-connection and out- connection matrices for the network system.

Therefore using this equation the discharge of the entire network system could be directly calculated.

## 4.0 Calculation Procedure for Network Systems

As the general discharge equation used herein is based on the Kinematic wave theory [8], which is again based on the theory that the characteristics traveling from the upstream point influences the down stream point [9] [10], the calculation order will have to be from upstream to downstream which is node 1, edge 1, node 2, edge 2 .... edge Ne, node Nn: where Nn is the number of nodes and Ne is the number of edges.

A descriptive process flow for the calculation procedure for the network system is as explained below.

1. Draw up the directed network system and thereafter using the method mentioned above deduce the optimum routing procedure.

2. Calculate the out-connection matrix $\bar{D}^+$ and the in-connection matrix $\bar{D}^-$ as in section 3.

3. Input space increment, time increment and channel data such as width, slope and Mannings coefficient etc.

4. Input initial condition for all reaches and nodes at time t=0.

5. Input boundary condition.

6. Calculate matrix elements $a_1$, ......, $a_9$ and $b_1$, $b_{m-1}$ of eq. 5.

7. Calculate values at the new time step using eq. 11 and eq. 13.

8. Repeat steps 6 and 7 until the desired time interval is reached.

## 5.0 Application of the Theory

The optimum routing procedure mentioned in section 2 is applied to a model network and the position after the execution of the relevant steps are as indicated in Fig. 1 - 4.

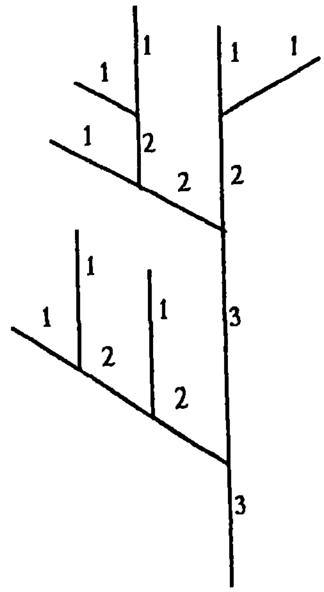


Figure 1

After execution of step-1
Numbers indicate the order of the reach

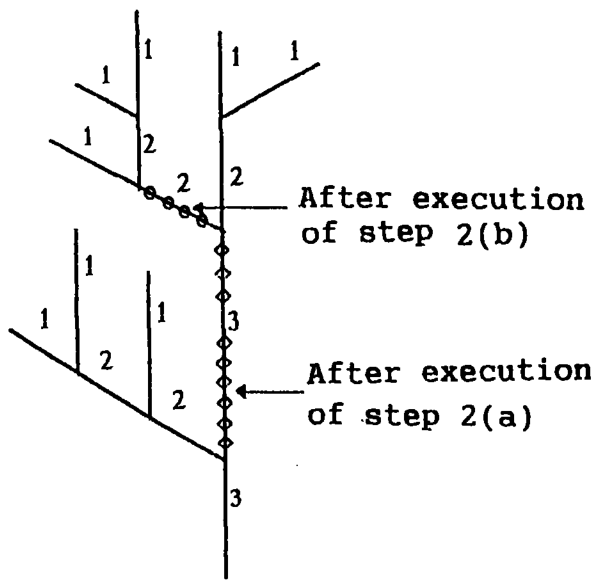


Figure 2

After execution of step-2
The marked lines indicate
the pivotel edge

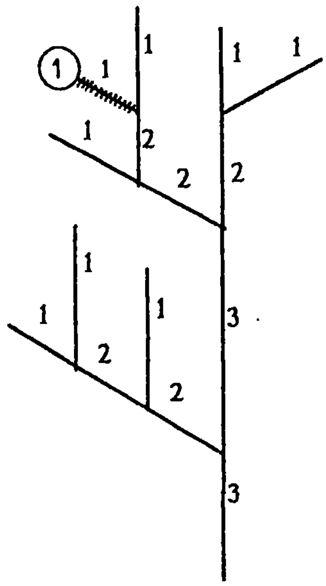

Figure 3

After execution of step-3
The marked line indicate the
edge first reached

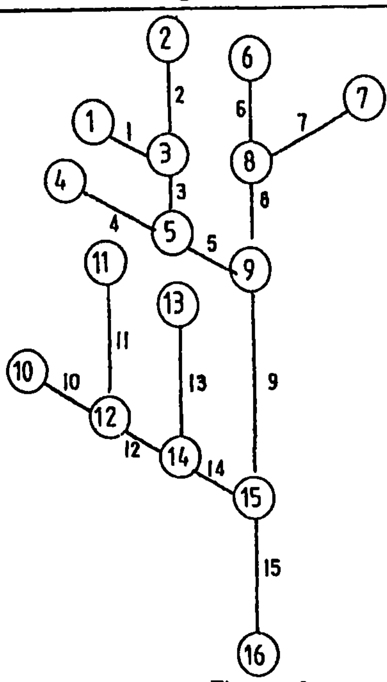


Figure 4
After execution of step-4
The numbers encircled indicate
the node number and the number
by the edge indicateS the edge number

Therefore, now that the optimum routing procedure of the network system has been found the in-connection matrix D and the out-connection matrix D for the entire network system could be calculated using equations 1 and 2.

$$\bar{D}^{+} =$$

$$\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

$$\bar{D}^{-} =$$

$$\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&-1&0&0\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1
\end{bmatrix}$$

Now inputting these data, the hydrological data, and following the calculation process mentioned in section 4, it is possible to route the network system directly.

## 6.0 Summary and Conclusion

a) A new simple 4 step procedure has been proposed for the optimum routing of channel network systems. The features of it are the simplicity and uniformity of the method.

b) A mathematical analysis of the network system has been carried out, and thereby a set of algorithms have been developed for the purpose of direct modeling of network systems and not piecewise modeling of channels.

## 7.0 References

[1] Takasao, T. and Shiba, M. : A Study on the runoff system model based on the topographical frame work of river basin, Journal of the Japan Society of Civil Engineers, Vol.240, pp.69-82, 1976.

[2] Minijiao, L.U., Hayakawa, H. and Koike, T. : Determination of optimal order of channel routing considering structure of channel network, Journal of The Japan Society of Civil Engineers, Vol. 473, No. 24, pp. 7-15, 1993.

[3] Strahler, A.H. : Hypsometric (area - altitude) analysis of erosional topography, Bull. of G.S.A., Vol. 63, pp. 1117 - 1142, 1952.

[4] Shimada, M. : Graph theoretical model for slow transient analysis of pipe networks, Journal of hydraulic Research, Vol. 115, No. 9, pp. 1165-1183, 1989.

[5] Kesavan, H.K. and Chandrashekar : Graph theoretic models for pipe network analysis, Proc. ASCE, Vol. 98, No. HY2, pp.345-364, 1972.

[6] Kanemoto, M. and Tsunematsu, Y. : Unsteady flow analysis of stream networks using Graph theory, Journal of The Japan Society of Civil Engineers, No. 429/2-15, pp.67-75, 1991.

[7] De Costa, S. : Kinematic routing model using the weighted residual method and its application to a Sri Lankan catchment, Proceedings of The 25th International Conference on Hydraulic Engineering, Vol.. 6, pp.41-48, 1993.

[8] Mishima, T., Kanamaru, A., Tsunematsu, Y. and De Costa, S. : Estimation of water holding capacity in hillside, Journal of the Japan Society of Hydrology and Water Resources, Vol. 6, No. 5, in press, 1994.

[9] De Costa, S., Tsunematsu, Y., Kanamaru, A. and Mishima, T. : Numerical analysis for Kinematic unsteady flows in stream networks of mountainous basins by means of the weighted residual method, Memoirs of The Fac. of Eng., Hiroshima Univ., Vol. 11, No.3, pp.77-85, 1993.

[10] Mishima, T., Kanamaru, A., Tsunematsu, Y. and De Costa, S. : Runoff mechanism of rainwater in hill side, Journal of the Japan Society of Hydrology and Water Resources, Vol. 6, No. 1, pp. 36-46, 1993.