# ACCELERATED SOLUTION OF THE FAST DECOUPLED NEWTON RAPHSON LOAD FLOW ALGORITHM IN POWER SYSTEM STUDIES

by

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## Abstract

The Fast Decoupled Newton Raphson Load Flow (FDLF) algorithm is one which gives the fastest solution to the Load Flow problem. However the convergence rate for the FDLF is geometric, while the Standard Newton Raphson achieves a quadratic convergence. The geometric nature of the convergence of the FDLF algorithm is exploited to extrapolate the solution towards faster convergence. The results of this study are presented in the paper and these show that a significant saving in computer time is achieved by the use of suitable accelerating factors for both monotonic as well as oscillatory convergence characteristics.

## List of Symbols

| | |
|---|---|
| A | - steady value of unknown x |
| $\alpha$ | - acceleration factor |
| B | - line susceptance matrix |
| Bi | - value of unknown x at iteration i |
| Ci | - deviation of unknown x from A at iteration i |
| f(x) | - function of x for which solution is sought |
| f'(x) | - derivative of f(x) with respect to x |
| J | - Jacobian matrix |
| n | - iteration count |
| P | - active power injected |
| $\Delta$P | - active power mismatch |
| Q | - reactive power injected |
| $\Delta$Q | - reactive power mismatch |
| r | - ratio in geometric progression |
| V | - magnitude of busbar voltage |
| $\theta$ | - phase angle of busbar voltage |
| $\Delta\theta$ | - increment of voltage angle |
| x | - general unknown variable |

## 1.0 Introduction

Load Flow analysis is an essential part in the planning and operation of a modern power system. Unlike in the early days, where the interest was in the development of algorithms and the economic use of computer storage, the present day interest is mainly on saving computer time. Of the common methods adopted for the solution of the load flow problem it has been established[1] that the Newton Raphson method has the lowest computer time for the solution of a network with a given bus size. The standard N-R algorithm has been subjected to a number of modifications whereby the speed has been increased and storage reduced. The improvement in the speed of solution of this fast decoupled sparsity oriented load flow (FDLF) algorithm[2] has been mainly in the calculation of mismatches using the Jacobian matrix

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and the use of sparsity techniques in the evaluations. The basic equation governing the solution in the Newton Raphson solution is equation (1.1).

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V/V \end{bmatrix} \quad \ldots\ldots (1.1)$$

The modifications incorporated into the Jacobian matrix (J) are two fold. Firstly, physical properties of the system dictate that the elements of the Jacobian matrix vary only slightly from iteration to iteration. Thus the amount of computation per iteration may be greatly reduced by forming the Jacobian as an invariant matrix and inverting it only once. Secondly, due to the highly reactive nature of the line impedance, the active power flow and reactive power flow calculations may be decoupled as these are dependant mainly on the voltage angle differences and voltage magnitude differences respectively. Thus the Jacobian matrix may be reduced to its 2 dominant diagonally placed sub-matrices. The basic equation (1.1) may now be re-expressed as that given by equation (1.2).

$$\begin{aligned} [\Delta P/V] &= [B][\Delta\theta] \\ [\Delta Q/V] &= [B][\Delta V] \end{aligned} \quad \ldots\ldots (1.2)$$

The [B] matrices appearing in equation (1.2) are the line susceptance matrices and are thus invariant.

These two modifications do not affect the final accuracy of the load flow solution, but the convergence characteristics are changed. While the Standard Newton Raphson algorithm has a quadratic convergence, the FDLF algorithm has only a geometric convergence. However, due to the removal of the necessity of the repeated evaluation of the inversion of the Jacobian matrix, and due to the decoupling, the solution time is reduced.

Previous studies have shown that the use of acceleration factors for the standard Newton Raphson algorithm do not cause any reduction in computer time. This is shown in Figure 1.1.

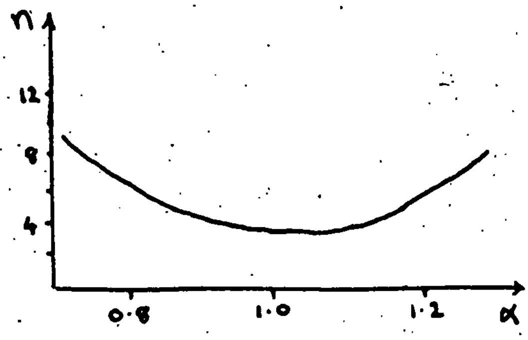


Figure 1.1

This has generally been assumed true even for the FDLF algorithm, so that acceleration factors have not been considered for this method in the past.

In this paper, studies carried out by the authors on the convergence characteristics of the voltage magnitudes and voltage angles are presented to show how suitable acceleration techniques give rise to faster convergence. The method is first illustrated by considering a simple example of the solution of a quadratic equation. Application to the FLDF algorithm is then discussed by considering the 14 bus IEEE Test System, and the 41 bus Sri Lanka Power System.

## 2.0 Newton-Raphson Method (N-R Method)

### 2.1 The Standard N-R algorithm

In the standard N-R method of solving a non-linear equation of the form $f(x) = 0$, an initial guess is made for x. This is repeatedly updated using equation (2.1) which has been derived from the Taylor's Series expansion.

$$\Delta Xn = - f(Xn) / f'(Xn)$$
$$Xn+1 = Xn + \Delta Xn \qquad \ldots\ldots \quad (2.1)$$

This technique is graphically explained in Figure 2.1, where convergence can be seen to be quadratic.

This may be graphically explained as shown in Figure 2.2, where the convergence is seen to be somewhat geometric, thus increasing the number of iterations.

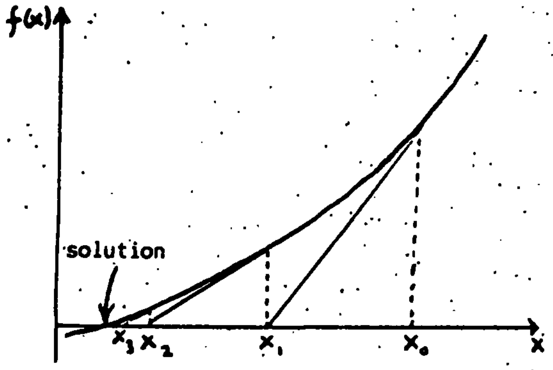

Figure 2.1

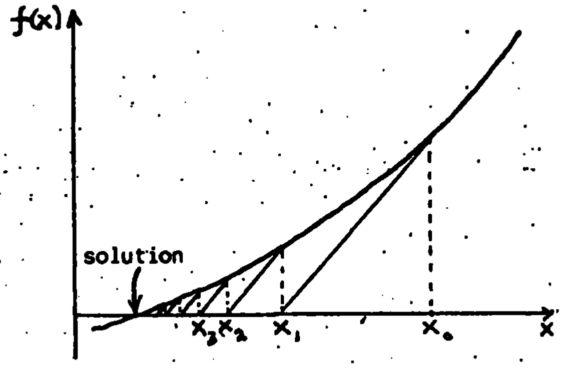

Figure 2.2

When a multi-variable set of equations is solved, the gradients correspond to a matrix of derivatives, referred to as the Jacobian matrix. Division by the derivative then corresponds to the inversion of the Jacobian Matrix. When the number of variables are large, it is very time consuming to repeat this operation at each iteration. Thus it is a common practice to use a modified form of the N-R algorithm.

$$\underset{\sim}{\Delta Xn} = - [J]^{-1}[f(\underset{\sim}{Xn})]$$
$$\underset{\sim}{Xn+1} = \underset{\sim}{Xn} + \underset{\sim}{\Delta Xn} \qquad \ldots\ldots \quad (2.2)$$

## 2.2 Modified N-R Algorithm

In the modified N-R method of solution, the derivative is calculated only at the initial guess. The subsequent updates are based on a constant gradient of this value, and the solution is given by the equation (2.3).

$$\Delta Xn = - f(Xn)/f'(Xo)$$
$$Xn+1 = Xn + \Delta Xn \qquad \ldots\ldots \quad (2.3)$$

In a single variable case, the time of solution will increase, due to the increase in number of iterations. However, in the multivariable case, a major part of the calculation is not repeated from iteration to iteration, as the Jacobian is needed to be inverted only once. Thus the total time of solution then infact decreases.

## 2.3 Accelerated Modified N-R Method

Since the use of a constant derivative (or a constant Jacobian) causes convergence not to be quadratic, and the iterations to increase, it is possible to accelerate the solution. This acceleration is achieved by extrapolation from values of the unknown at the earlier iterations.

For example, the convergence of the solution in the modified N-R method may be either monotonic (such as in Figure 2.3) or oscillatory (such as in Figure 2.4). If the type of solution is known, extrapolation may be used to give faster convergence.

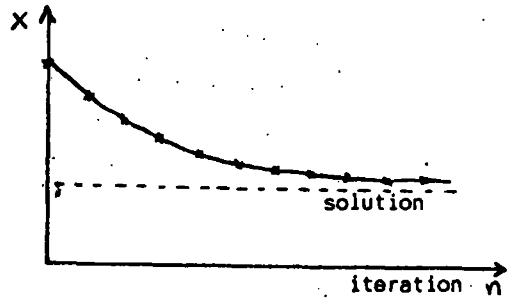


Figure 2.3

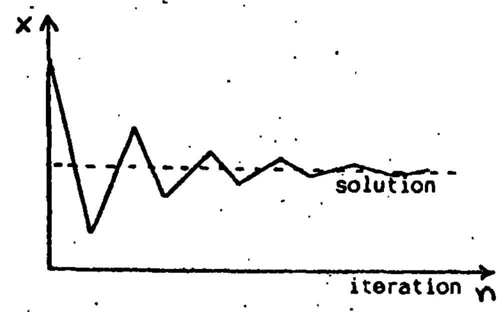


Figure 2.4

## 3.0 Selection of Acceleration Factors

If $\alpha$ is the acceleration factor to be applied, then an approximate solution may be improved using the calculated increment and the acceleration factor as follows.

$$Xnew = Xold + \alpha . \Delta x$$

In the case of monotonic convergence, faster convergence can be obtained using values of $\alpha$ greater than 1. Typical values are in the range 1.0 to 2.0. On the other hand for oscillatory type of solution, suitable acceleration factors should be less than 1, as each increment causes overshooting. Typical acceleration factors in this case are in the region 0.5 to 1.0.

For methods such as Gauss-Seidel, the general practice is to select a constant value of acceleration factor of around 1.5 without determining the nature of the convergence. For the modified N-R algorithm which is used in the FDLF, the acceleration factors of this order are generally too high. Also, it is important to determine the nature of convergence.

Although it was mentioned that the modified N-R algorithm has a geometric convergence, this is not exactly so. The convergence characteristic also depends on the degree of the non-linearity as well as on the inter-dependancy of the variables. In this study, the extrapolations are based on the assumption of geometric convergence. Thus final values cannot be predicted exactly in one stage of extrapolation.

### 3.1 Monotonic Convergence

In the case of geometric monotonic convergence, shown in Figure 3.1, the deviations Ci from the final value A are in geometric progression with positive ratio r.

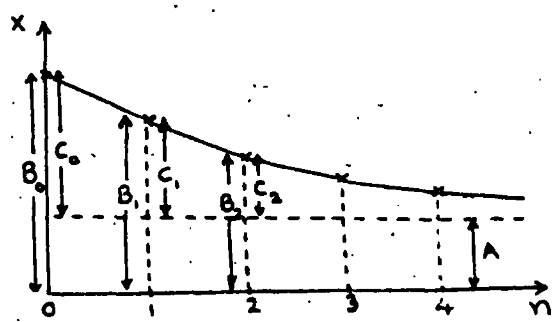


Figure 3.1

Thus C1 = r Co, C2 = r C1 = r² Co, ...... and so on.

The calculated values of the unknown, Bi, are related to the deviations by Bi = Ci + A.

If two adjacent iterations are considered, we may write

Bi = Ci + A, Bi + 1 = Ci + 1 + A, Ci + 1 = r Ci

i.e. Bi + 1 = r Ci + A

similarly $B_{i+2} = r\,C_{i+1} + A$

from which $r = \dfrac{B_{i+2} - B_{i+1}}{B_{i+1} - B_i} = \dfrac{\Delta B_{i+1}}{\Delta B_i}$

also, $B_{i+1} - r\,B_i = (1 - r)\,A$

Thus $A = (B_{i+1} - r\,B_i)/(1 - r) = B_i + (B_{i+1} - B_i)/(1 - r)$

The final value may thus be estimated as

$$A = B_i + \Delta B_i / (1 - r)$$

where $\alpha = 1/(1 - r)$ is the acceleration factor.

### 3.2 Oscillatory Convergence

In the case of geometric oscillatory convergence, shown in Figure 3.2, the deviations $C_i$ from the final value A are again in a geometric progression but with negative ratio r.

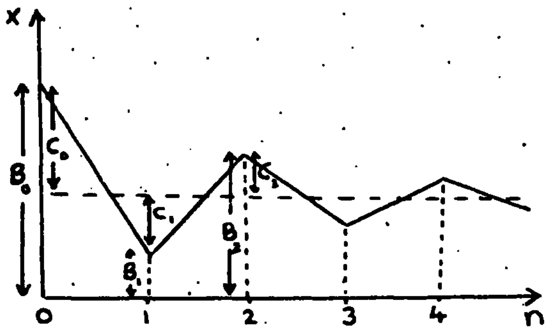


Figure 3.2

The final value may again be obtained from

$$A = B_i + \Delta B_i/(1 - r)$$

### 3.3 Practical convergence

In the case of the modified N-R method used in the FDLF solution, the convergence for the first few iterations quite often differs in form from the regular convergence thereafter (whether it be monotonic or oscillatory). This is because of the flat initial start. Typical convergence characteristics are shown in Figure 3.3.

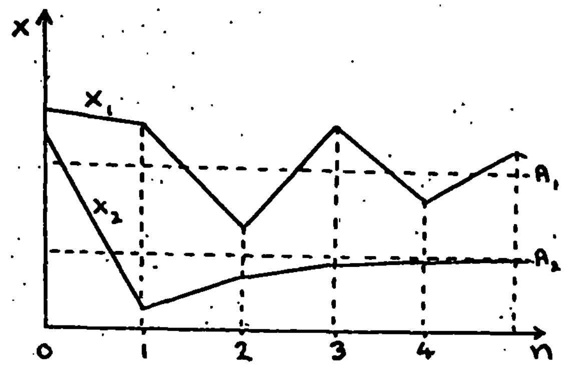


Figure 3.3

Thus the acceleration factors, which aim at the final value, should be applied after avoiding the initial region of non-regular behaviour. Also, in practice, once the acceleration factors have been applied, the convergence characteristic becomes disturbed and no longer regular for the next few iterations. These regions too are to be avoided when applying extrapolation.

## 4.0 Case Studies

### 4.1 Case Study 1 - Solution of quadratic equation $f(X) = 4X^2 - 4X - 15 = 0$

Figure 4.1 shows the variation of the function f(x) with x.

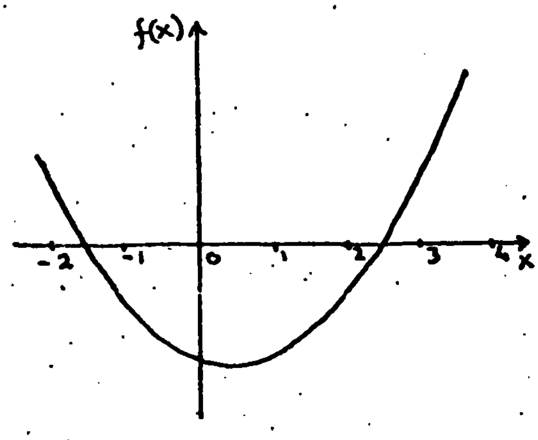


Figure 4.1

The solution is obtained by three methods, namely, (i) the N - R algorithm, (ii) the modified N - R algorithm and (iii) the accelerated modified N - R algorithm with extrapolation based on geometric convergence.

To illustrate the advantage of acceleration with the modified N - R algorithm, three values of initial guesses are considered, and the convergence characteristics investigated. The results for the 3 initial guesses are shown in Figures 4.2, 4.3 and 4.4.

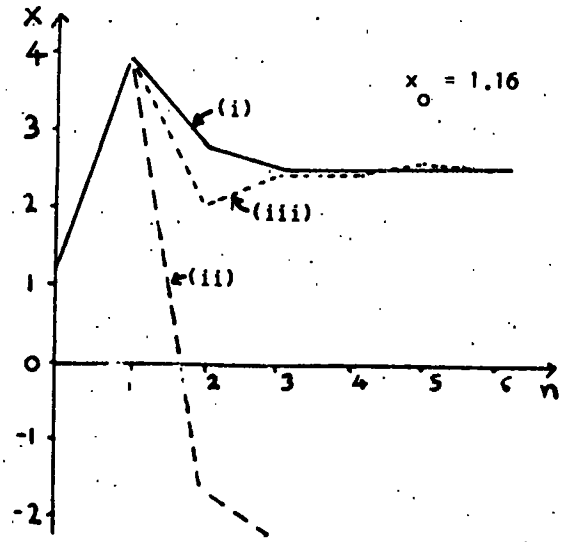


Figure 4.4

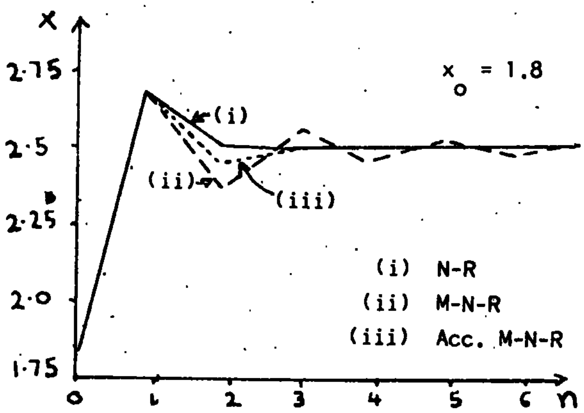


Figure 4.2

It is clearly seen that the N - R solution converges very rapidly for each of the 3 guesses. However, if the modified N - R method is used, convergence is obtained only if the initial guess is quite close to the final solution (as in xo = 1.8), where as it either oscillates (with xo = 1.2) or diverges (with xo = 1.16) as the initial guess becomes less accurate. It is seen that with the use of the acceleration factor causes all 3 cases converge rapidly, although not as fast as the N - R solution.

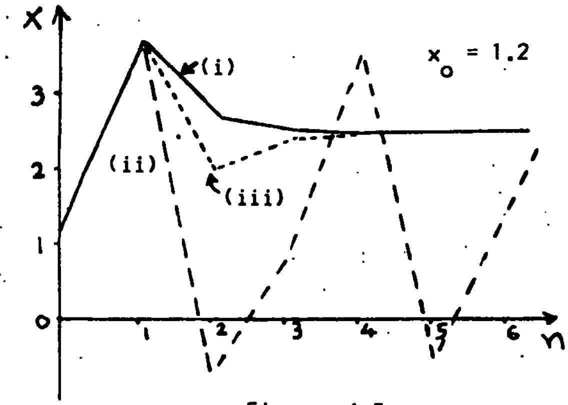


Figure 4.3

## 4.2 Case Study II - Load Flow Study on the 14-bus IEEE test system

The convergence characteristics for the 14-bus IEEE test system was obtained with the FDLF algorithm, without acceleration applied. The voltage magnitude variations and voltage angle variations for selected nodes are shown in Figures 4.5 and 4.6 respectively. (Information for all 14 nodes are not shown, to improve the clarity of those shown).

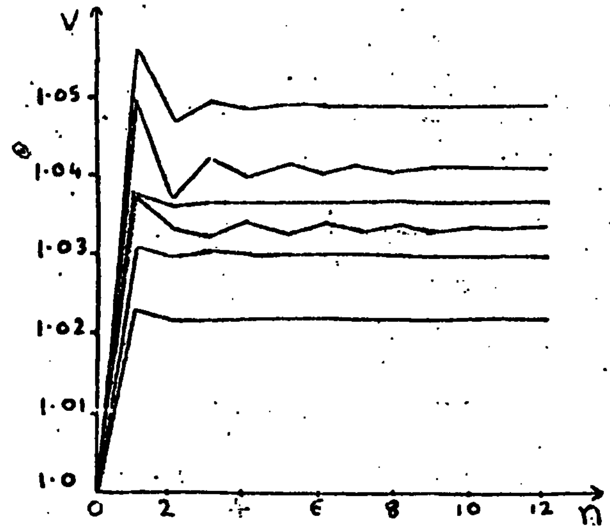


Figure 4.5

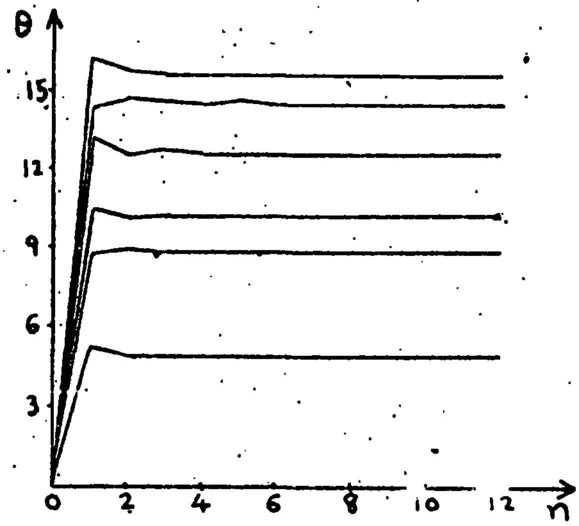


Figure 4.6

The variations of these selected voltage magnitudes and voltage angles are shown to a logarithmic base in figures 4.7 and 4.8 respectively.

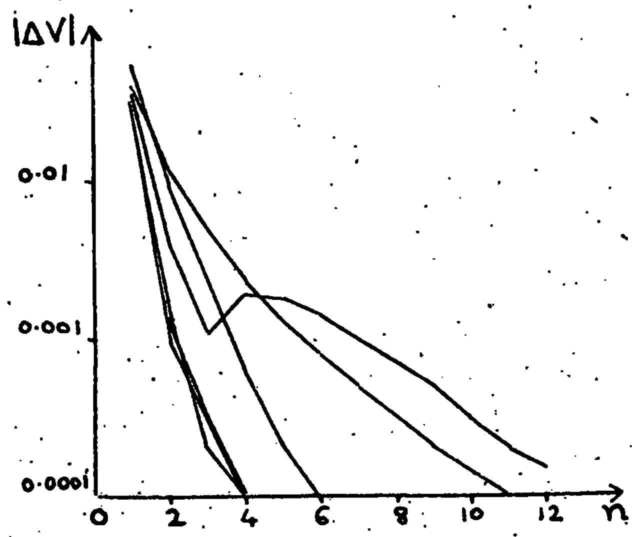


Figure 4.7

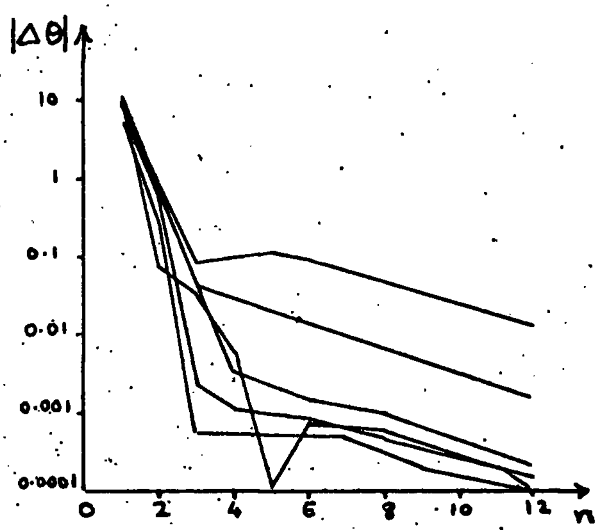


Figure 4.8

Analysis of these figures show that suitable acceleration factors for monotonic convergence and oscillatory convergence are 1.275 and 0.820 respectively. Acceleration factors corresponding to these values were applied and the convergence characteristics again obtained. These are shown in Figures 4:9 and 4.10 for voltage magnitude and angle respectively.

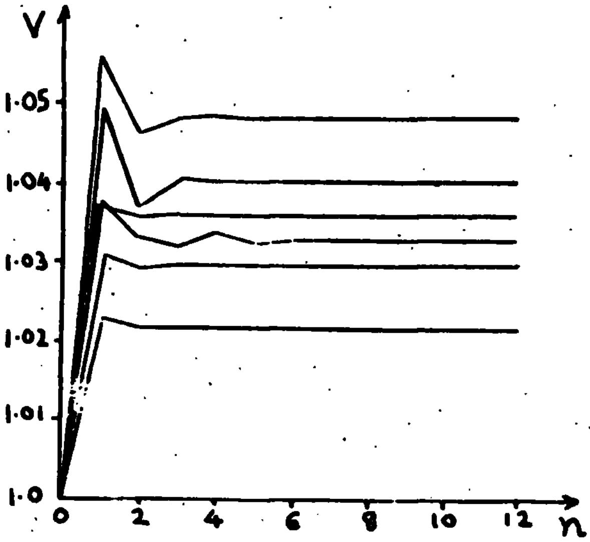


Figure 4.9

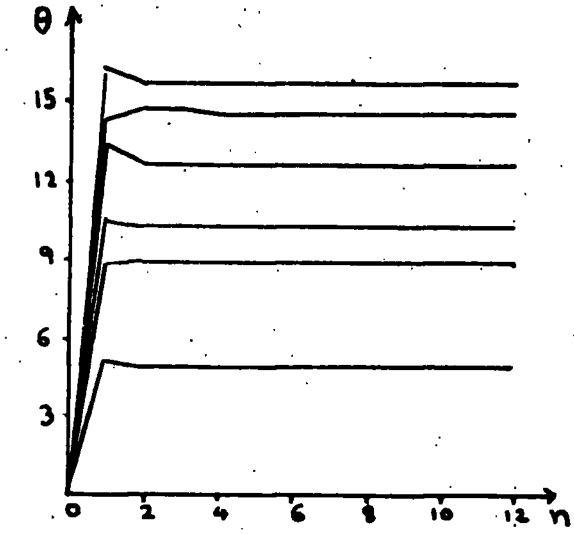


Figure 4.10

It is seen that the solution converges much faster with the acceleration factors applied; the number of iterations reducing from 12 to 6 on the SORD M685 computer.

The variation of the maximum power mismatch with and without acceleration are shown for the active power flow and the reactive power flow in Figures 4.11 and 4.12 respectively.

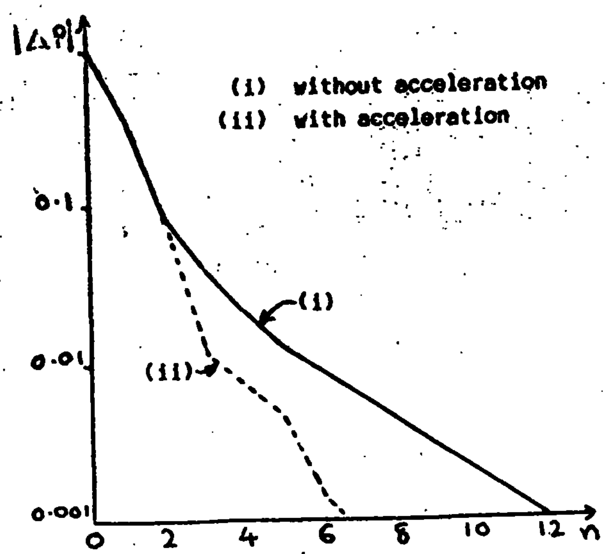


Figure 4.11

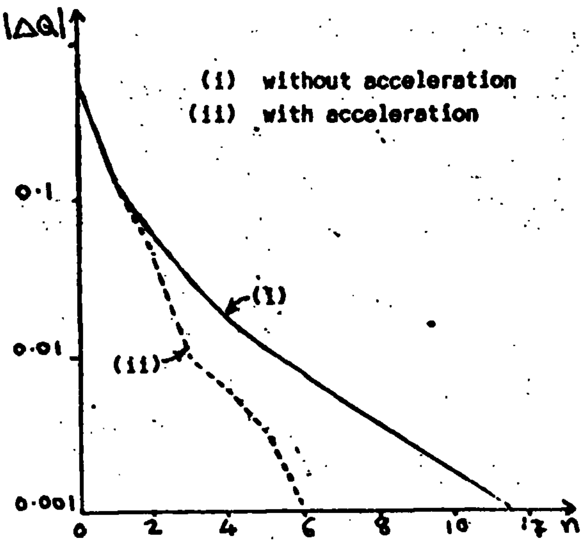


Figure 4.12

### 4.4 Case Study III - Load Flow Study on the Sri Lanka Power System

Convergence characteristics of the Sri Lanka Power System was considered for a 41 bus configuration. The variation of the maximum power mismatch with and without acceleration are shown for the active power flow and the reactive power flow in Figures 4.13 and 4.14 respectively.

structures and contain no reference to the formation of the communal village tanks. Under the above circumstances we have nothing to prove the age of works except the evidence obtainable from dimensions of bricks. In addition, literature on ancient irrigation works during recent past was mostly a qualitative analysis and therefore it is extremely difficult to get information required for quantitative analysis:

Figure 4.13

4. The special feature of the ancient civilization of Ceylon was its irrigation works. Different systems of irrigation were adopted. The water [illegible] in 1858. In his famous minute he described as one of the noble monuments of ancient enterprise and science he had seen".2/

5. There had been three main diversion weirs during Pre Christian Era across following rivers.

a. Malwathu Oya - to augment Nuwara Wewa
b. Kirindi Oya - to augment Tissa Wewa
c. Kal Aru - to augment some village tanks.

6. According to, the special point of interest in Kirindi Oya dam was the astonishing fact that instead of being taken across the river by the shortest possible line as one would expect, it was built at an oblique angle. "The knowledge of which was only acquired in comparatively recent years in Europe was understood in Ceylon in very early times".1/

7. According to Mahawansa construction of tanks in Ceylon had commenced in the first half of 400 B.C. and parameters of 10 tanks during this period are given in Table 1 for comparison. Table (1) shows the relevant parameters of ten tanks constructed from 400 B.C. to 300 A.D.

8. It is evident from this table that in present day designs we adopt almost 3 times of spill length in comparison to what was provided by ancient builders. In order to compensate for narrow spill lengths very high free boards had been adopted in ancient designs. Nachchaduwa, Nuwara Wewa, Pawatkulam, Basawakkulam and [illegible] had been breached and there had been three breaches. Duratissa Wewa (Yodawewa) in Southern Province constructed by Sadda Tissa in 100 B.C. was repaired by Parakrama Bahu 1 in 11th century A.D. indicating the difficulty to find out the exact age of original construction due to subsequent repairs.

Figure 4.14

9. The most important feature of these tanks to our present day designers is the existence of very large free board in the ancient design. When restoring an ancient tank one may jump to the conclusion that original water level of the ancient tank had been near the existing bund top level. Due to non-availability of data regarding the original spill elevation, we may design the full supply level of the new tank near to the original bund top level. I wonder whether Kantale disaster is also due to this type of wrong


'Engineer', September/December 1988 (13)